EXERCISE 1

1.2) Possible mathematical model (BLP)

$$x_j = \begin{cases} 1 & \text{if item } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \dots, n$$

$$\min \quad z = \sum_{j=1}^{n} c_j x_j$$

s.t.

$$\sum_{j=1}^{n} p_j x_j \geq a \qquad (a)$$

$$\sum_{j=1}^{n} x_j \geq b \qquad (b)$$

$$x_j \in \{0, 1\} \qquad j = 1, \dots, n$$

1.1) Size of P: $n, a, b, (c_j), (p_j) \Rightarrow O(n)$: n

- $P \in NP$ (decision tree with n levels, 2 descendent nodes)
- KP01-min $\propto P$ (KP01-min: $\bar{n}$, $\bar{b}$, (P_j), (W_j) size $\bar{n}$)

$n := \bar{n}$; $a := \bar{b}$; $c_j := P_j$, $p_j := W_j$ $j = 1, \dots, \bar{n}$ $\quad O(\bar{n})$

$b := 0$

$\Rightarrow P \in NP\text{-Hard}$

1.3.1) $F\text{-}P \in \mathcal{P}$ (set $x_j := 1$ for $j = 1, \dots, n$; check if (a) and (b) are satisfied) $\quad O(n)$

1.3.2) $F\text{-}P \in \mathcal{P}$ (1. sort the n items according to non-increasing values of p_j;

2. set $x_j := 1$ for $j = 1, \dots, b$; $x_j := 0$ for $j = b+1, \dots, n$
3. check if (a) is satisfied)

$O(n \log n)$

1.3.3) $F\text{-}P \in \mathcal{P}$: as done for 1.3.2).

1.3.4) $F\text{-}P \in NP$ (...)

$PP \propto F\text{-}P$ (... ; $b := 0$) $\Rightarrow F\text{-}P \in NP\text{-Hard}$

Exercise 2.

$n, m, (a_j), (b_j), c$

∘ grandezza: $2n+3 \to O(n)$

$(m < n)$

2a) • $P \in NP$ (albero decisionale di n livelli: 1 per operazione) al massimo m nodi figli (1 per macchina)

$(PP$ con $\bar{c} = \sum_{j=1}^{n} p_j / 2) \; \alpha \; P$: $n := \bar{n}$, $c := \bar{c}$, $a_1 = 0$, $b_1 = p_1$; $a_j = b_{j-1} + 1$, $b_j = a_j + p_j$

$PP(\bar{n}, (p_j), \bar{c})$ per $j = 2, \dots, n$; $m = 2$. PP ammette soluzione

Grandezza: $\bar{n}$ $O(\bar{n})$ se e solo se P ammette solu

Quindi $P \in NP$-Hard

2b)

$$y_i = \begin{cases} 1 & \text{se macchina } i \text{ utilizzata} \\ 0 & \text{altrimenti} \end{cases} \qquad i = 1, \dots, m$$

$$x_{i,j} = \begin{cases} 1 & \text{se operazione } j \text{ eseguita da macchina } i \\ 0 & \text{altrimenti} \end{cases} \qquad i = 1, \dots, m; \; j = 1, \dots, n$$

$$\min \; z = \sum_{i=1}^{m} y_i$$

s.t.

$$\sum_{i=1}^{m} x_{i,j} = 1 \qquad j = 1, \dots, n$$

$$\sum_{j=1}^{n} (b_j - a_j) x_{i,j} \le c \, y_i \qquad i = 1, \dots, m$$

$$x_{i,j} + x_{i,k} \le 1 \qquad i = 1, \dots, m; \; j = 1, \dots, n; \; k \in S_j$$

$$x_{i,j} \in \{0, 1\} \qquad i = 1, \dots, m; \; j = 1, \dots, n$$

$$y_i \in \{0, 1\} \qquad i = 1, \dots, m$$

con:

$S_j := \{k : \text{operazione } k \text{ si sovrappone ad operazione } j\} \quad j = 1, \dots$