Recent Developments in Location-Routing Problems

Deterministic, single-echelon, single-objective, single-period problems

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Abstract

Location-routing problems (LRPs) simultaneously decide on the locations of facilities and routes originating from these facilities to serve a set of customers. This survey describes recent developments in deterministic, single-objective LRPs with a single echelon and a single planning period. We give detailed summaries of the papers, focusing on the central properties of the described problems and the proposed solution methods. We identify shortcomings in the current literature, provide recommendations to mitigate them, highlight application cases, and list promising topics for further research.

Keywords:location-routing problem; survey; deterministic problems

1 Introduction

Location-routing problems (LRPs) are a class of optimization problems in transportation and logistics that simultaneously optimize the following two key decisions:

1. Location decisions: choosing a set of facilities (e.g., warehouses or depots) to open. We use the term "facility" throughout the paper independently of the type of object to be located. The facilities are chosen from a set of potential locations to minimize the cost of satisfying customer demands or meeting other criteria such as minimizing emissions or maximizing profit.

2. Routing decisions: devising routes for a fleet of vehicles to serve customers from the open facilities. These decisions involve choosing which customers each vehicle should visit and the sequence of these visits.

Although the location and routing decisions are optimized jointly, in practical applications, the former are usually strategic and tactical decisions taken at mid-to-long-term time horizons, while the latter are operational. As pointed out by Said Salhi and Nagy (1999), considering these different decisions together leads to superior solutions in many applications. The majority of LRP applications arise in the field of logistics, especially in goods distribution, postal services, and waste collection. Still, LRP models have shown exceptional adaptability and have been used in fields such as military operations, space exploration, public transit planning, and disaster relief.

Our definition of what constitutes an LRP extends the guideline introduced by Drexl and Schneider (2015) and Schneider and Drexl (2017). We focus on problems in which the facilities are the starting and/or ending points of routes. Therefore, we exclude LRPs with intermediate stops for refueling, recharging, or reloading vehicles (see, e.g., Schiffer et al. 2019). Furthermore, we focus on problems in which the routing decisions do not implicitly determine location decisions. The problems we consider arise, for instance, if a cost is incurred for opening a facility, a limited number of facilities must be selected, or the facilities are capacitated. By contrast, when opening a facility incurs no cost, or one can open any number of uncapacitated facilities, solving a routing problem is sufficient to make the location decisions. For this reason, we do not consider, e.g., multi-depot vehicle routing problems (VRPs, Cordeau et al. 1997), VRPs with intermediate depots or refill points (Tarantilis et al. 2008), pickup-and-delivery problems with transshipments (Rais et al. 2014), and VRPs with trailers and transshipments (Drexl 2012). Furthermore, we exclude problems without location decisions, such as vehicle routing-allocation (also known as median cycle) problems (Labbé et al. 2005) and Hamiltonian p-median problems (Branco and Dias Coelho 1990). We also exclude problems with no routing decisions, such as pure facility location problems (Fernández and Landete 2019) and service network design problems (Crainic and Kim 2007). LRPs are also similar to districting problems because both typically involve an allocation decision (Kalcsics and Ríos-Mercado 2019), however, the latter are also excluded from this survey.

LRPs have been studied for over 40 years and have received a lot of attention in the last 25 years. The more recent surveys include Nagy and Saïd Salhi (2007), Lopes Borges et al. (2013), Prodhon and Prins (2014), Drexl and Schneider (2015), Cuda et al. (2015), Schneider and Drexl (2017), and Albareda-Sambola and Rodríguez-Pereira (2019). Recently, Mara et al. (2021) have presented a taxonomy of LRPs, and Marinakis (2025) provides an encyclopedia-style introduction to LRPs. Multi-objective problems have been reviewed by Tadaros and Migdalas (2022) and hub LRPs by Wandelt et al. (2025).

The goal of this paper is to provide a broad overview of the progress in the field of LRPs since the full-fledged surveys of Prodhon and Prins (2014), Drexl and Schneider (2015), and Schneider and Drexl (2017), following the idea of the latter two surveys to provide rather detailed descriptions of most included papers. To keep the amount of material manageable, we have to restrict the scope of our survey to deterministic LRPs. In this paper, we describe recent developments in single-objective, single-echelon, and single-period LRPs. In a follow-up paper, we treat multi-objective, multi-echelon, and multi-period problems. The intended audience of this survey are scientists and practitioners with previous experience in location routing. However, to help readers approaching this topic for the first time, in Section A of the online companion, we include mathematical models for what are, in our opinion, the most relevant LRP variants.

The survey is restricted to published full-length papers (journal publications or conference proceedings) not already included in the surveys by Drexl and Schneider (2015) and Schneider and Drexl (2017) or in earlier surveys. Thus, we exclude PhD and Master's theses, working papers, and technical reports. Our goal is not to cover the entire literature but to point to the most important works according to our subjective opinion. To this end, we analyzed over 250 papers that fulfilled the above criteria and selected over 120 to be included in this paper. As mentioned above, our paper summaries are rather detailed because we want to provide the reader with an understanding of the papers' main modeling and algorithmic ideas. The lengths of the descriptions vary and depend on many factors such as the methodological complexity, novelty, and length of the original paper, as well as our own subjective opinion about its importance to the field.

More specifically, Section 2 reviews standard LRPs, defined by Schneider and Drexl (2017) as a deterministic, discrete, single-echelon, single-objective, single-period problem in which no inventory decisions are relevant, and each customer must be visited exactly once from a single facility by a single vehicle. In Section 3, we cover LRPs with pickup and delivery (LRPPDs), in

Name	Reference	Type	#instances	#facilities	#customers
Tuzun	Tuzun and Burke (1999)	Standard LRP	36	10–20	100–200
Prins	Prins et al. (2006)	Standard LRP	30	5–10	20–200
Barreto	Barreto et al. (2007)	Standard LRP	13	5–14	21–150
Duhamel	Duhamel et al. (2010)	Standard LRP	30	2–23	5–164
Akca	Akca et al. (2009)	Standard LRP	12	5	30–40
Schneider	Schneider and Löffler (2019)	Standard LRP	202	5–30	100–600
Harks	Harks et al. (2013)	Standard LRP	27	100–1000	1000–10000
Ponboon	Ponboon et al. (2016)	LRPTW[1]	29	3	10–40

Table 1: Benchmark instances. [1]LRP with time windows, see Section 5.

which customers can require both delivery and pickup of goods. Section 4 considers LRPs with profits (LRPPs), i.e., a class of problems arising when not all customers must be served, and the planner must consider the tradeoff between facility and route costs and the profits earned by serving customers. In Section 5, we survey LRPs with time windows (LRPTWs), in which the service at customers must start within a specific time interval. Section 6 discusses latency LRPs (LLRPs), which assume that customers prefer to be visited as early as possible. Finally, Section 7 groups LRP variants that, while being single-period and single-objective, do not fit in any of the previous categories. Note that if a paper addresses a problem whose assignment to a known class of LRPs is not unique, e.g., a problem features pickup and deliveries and its objective is to maximize profits, we discuss it with the class that, according to our opinion, represents the more defining feature of the problem. In Section 8, we identify shortcomings in the current literature, provide recommendations to mitigate them, highlight application cases, and list promising topics for further research.

Finally, because some papers compare their proposed methods on benchmark instance sets, we briefly introduce these sets in Table 1. Irrespective of the names and abbreviations used previously in the literature (sometimes inconsistently), we refer to the instance sets using the name of the first author of the paper that introduced them to increase readability. The table lists all instance sets used in at least two papers. We collected the instances available online as of February 2025 in a GitHub repository to make them permanently and effortlessly available to the research community (Cavagnini et al. 2025).

2 Standard location-routing problems

In this section, we survey the literature on the standard LRP as defined in Section 1 (for a mathematical formulation, see Appendix A.1 in the online companion). Figure 1 shows the solution of a standard LRP. Depending on the application, standard LRPs may have (un)capacitated facilities, (no) fixed costs for opening a facility, and (un)limited/(un)capacitated fleet. Because most of the reviewed papers consider capacitated facilities with fixed opening costs and capacitated vehicles, we only indicate exceptions from this case.

In the open LRP (OLRP)—which falls under our definition of standard LRPs—vehicles are not required to return to the facility at the end of their routes, e.g., if some routes are performed by occasional drivers who use their own vehicles, as is common for last-mile delivery companies like Uber Eats or Deliveroo (see, e.g., Martínez-Sykora et al. 2024). Figure 2 shows the solution of an OLRP. The OLRP was introduced by V. Yu and S.-Y. Lin (2015a), who presented a MIP formulation and a simulated annealing (SA) algorithm. We note that OLRPs can be tackled with models and heuristics for the standard LRP by adjusting the input data of the model so that the cost of all customer-facility arcs is set to zero.

We group the papers according to methodology: exact methods (Section 2.1), heuristic methods (Section 2.2), and approximation methods (Section 2.3). Table 2 in the online companion

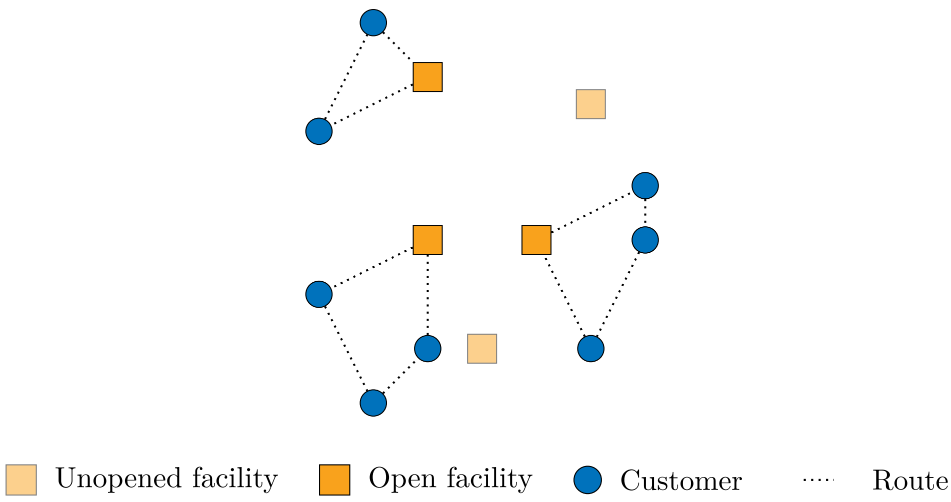

Figure 1: Example solution of a standard LRP.

summarizes the main characteristics of the surveyed papers on standard LRPs.

2.1 Exact methods

Since the last survey of Schneider and Drexl (2017), only Liguori et al. (2023) have proposed an exact method. The authors develop a branch-cut-and-price (BCP) algorithm based on an extended formulation that includes an exponential number of route variables. The main contribution is the introduction of route load knapsack cuts (RLKCs), a family of nonrobust valid inequalities, i.e., inequalities that change the nature of the pricing subproblem. Each RLKC identifies one or more subsets of routes starting at the same facility that, taken together, would exceed the capacity of the facility. The RLKCs do not use route variables directly but rely on integer variables that count how many selected routes have a given onboard load. For example, if a facility has a capacity of 10, an RLKC could impose that at most one route starting at that facility is selected among all routes with an onboard load of 6 or 7. The pricing subproblem is solved using a labeling algorithm. Using polyhedral and algebraic properties of the RLKCs, the authors show that their dual values can be used without substantial modifications to the labeling algorithm, even when incorporating advanced techniques such as bidirectional labeling. This property is fundamental to the effectiveness of the proposed BCP, and computational experiments show that this algorithm qualifies as the state-of-the-art exact algorithm for solving the standard LRP. The BCP solves 11 instances from the Tuzun and Prins sets that could not be solved by Contardo, Cordeau, et al. (2014). On the Schneider instance set, the BCP solves to optimality all instances with 100 customers, three quarters of the instances with 200 customers, one quarter of the instances with 300 customers, and improves the best known solutions (BKSs) in Schneider and Löffler (2019) for 62 instances. The results obtained on instances with 300 customers and 20 depots show that there is still improvement potential in the state-of-the-art heuristics discussed below.

Granada et al. (2019) present a new formulation for OLRP. They observe that an OLRP solution is a spanning forest consisting of one tree per open facility and propose a mixed-integer program (MIP) exploiting this graph structure using constraints ensuring that: (i) the number of selected arcs is equal to the number of customers, (ii) exactly one arc enters and leaves each customer except for those at the end of a route, and (iii) the flow is balanced, i.e., customers are fully served when visited, and the vehicle is empty after serving the last customer. The authors solve the Tuzun, Barreto, and Prins sets interpreted as OLRP instances using CPLEX with a four-hour time limit and compare the quality of their solutions to the one obtained with the model of V. Yu and S.-Y. Lin (2015a): the new formulation finds the same optimal solutions in

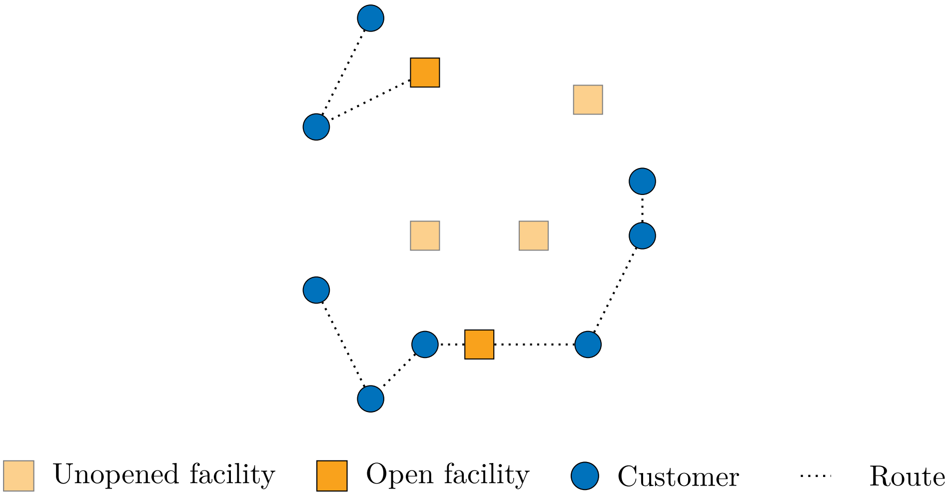

Figure 2: Example solution of an OLRP.

shorter runtimes, finds several additional optima, and provides stronger upper and lower bounds for all instances. Several new BKSs are reported compared to the previous results of V. Yu and S.-Y. Lin (2015a) and Schneider and Löffler (2019).

2.2 Heuristic methods

Arnold and Sörensen (2021) develop a progressive filtering (PF) heuristic for the standard LRP that eliminates as many facility configurations as possible before evaluating them using a routing phase. PF first removes all facility configurations with more facilities than the maximum between: (i) the minimum number of facilities required to satisfy customer demand (if dealing with facility capacities), and (ii) the minimum number of facilities that should be opened such that the estimated maximal benefit of adding another facility is less than the opening cost of the least expensive facility, as determined via a computational study. If the obtained upper bound is small, a complete enumeration of facility configurations is performed. Otherwise, promising configurations are generated via a heuristic that prioritizes facilities that are located in areas with a high customer density and have low opening costs and short distances to customers. In the routing phase, starting solutions are generated with a regret-based savings heuristic (Clarke and Wright 1964) and further improved by the knowledge guided local search heuristic (KGLS) of Arnold and Sörensen (2019). This local search (LS) uses a CROSS-exchange operator (Taillard et al. 1997) that swaps two substrings of customers belonging to two different routes, and a relocation chain (Rego 2001) that executes up to three sequential relocate moves, each involving the transfer of a single customer to a different route. The LS is based on a best-improvement search strategy and accepts only feasible moves. In each LS iteration, if the best move improves the solution, the affected routes are reoptimized with the heuristic of S. Lin and Kernighan (1973). As soon as a local minimum is reached, a perturbation phase with a given number of iterations is applied. In each iteration, a guiding mechanism increases the cost of a used edge that is selected according to one of the following criteria: (i) the longest edge, (ii) the farthest edge from the line connecting the route's center of gravity to the facility, and (iii) the edge with the biggest sum between length and distance from the line connecting the route's center of gravity to the facility. Then, the previously described LS is applied using this modified cost to evaluate the moves. After the perturbation phase, another run of LS uses actual edge costs. Increasingly accurate versions of this KGLS with more iterations are used to eliminate more and more facility configurations. On the Tuzun, Barreto, and Prins instance sets, PF dominates all state-of-the-art algorithms from the literature except for specific versions of the algorithms of Hemmelmayr et al. (2012) and R. B. Lopes et al. (2016) and the tree-based search algorithm

(TBSA) of Schneider and Löffler (2019). On the Schneider instance set, PF outperforms TBSA, especially because it consistently obtains solutions with lower total cost than those produced by TBSA for instances with a large number of possible high-quality facility configurations. On the very large-scale Harks instance set, PF finds solutions superior to those of the approximation algorithm of Harks et al. (2013) for almost all instances. PF also successfully solves instances of the two-echelon LRP (2ELRP; see, e.g., Drexl and Schneider 2015).

The iterated LS (ILS) of Accorsi and Vigo (2020), called AVXS, is designed for different single truck and trailer routing problems. However, it can also be used to solve instances of the LRP with uncapacitated facilities like in the Tuzun set. The entire ILS is embedded in an outer multistart framework. In each restart iteration, the assignment of customers to facilities is modified, and, for each facility, a capacitated VRP (CVRP) is solved using the Clarke and Wright (1964) savings algorithm to generate a starting solution for the ILS. The perturbation is based on a ruin-and-recreate mechanism, and a variable neighborhood descent (VND) with a randomized order of the neighborhood operators is used as LS. During the ILS execution, high-quality routes are stored in a solution pool and later used in a postprocessing step that solves a set partitioning problem. On the Tuzun set, AVXS is competitive with TBSA and PF.

Löffler et al. (2023) present a hybrid of greedy randomized adaptive search procedure (GRASP) and variable neighborhood search (VNS) for the standard LRP and study the influence of each of the algorithmic components on the solution quality and runtime. The algorithm starts with a GRASP phase, in which the randomized extended Clarke and Wright savings algorithm (RECWA, see Prins et al. 2006) is applied to determine the initial facility configuration and vehicle routes. To improve the quality of the routing solutions, the authors replace the greedy LS of the original RECWA with a more powerful VND that uses twelve neighborhood operators and can handle infeasible solutions with regard to vehicle and/or facility capacities. The three best solutions found in this GRASP phase are stored, and a VNS is applied to improve their routes. The VNS uses the above-described VND as LS and a cyclic-exchange operator (Ibaraki et al. 2005) in the shaking phase. The authors note that the suboptimal routes that are used to assess the quality of the facility configurations in the GRASP in tendency lead to too many open facilities. Therefore, a refinement phase evaluates solutions with one fewer facility than the configuration of the best-found solution, using a short VNS run to improve the routes of these configurations. Finally, the best-found solution undergoes a finalization phase in which a longer VNS run is used to improve routing costs. The results of the computational experiments performed on the Barreto, Tuzun, and Prins instance sets show that the heuristic is not completely competitive with the state-of-the-art algorithms of Schneider and Löffler (2019) and Arnold and Sörensen (2021), but shows a decent performance for such a conceptually simple heuristic. Each of the implemented algorithmic components has a positive impact on the solution quality. In particular, the finalization phase is vital (even if it causes higher runtimes), and the facility configuration refinement has more impact for instances with high facility costs compared to routing costs.

Voigt et al. (2022) use an adaptive large neighborhood search (ALNS) within a genetic algorithm (GA) framework to solve the standard LRP. To generate the initial population, an ALNS is executed for several runs, each of which generates a solution. The candidate customers for removal of all destroy operators are determined by comparing the current solution to the best found so far (also covering all previous runs). If the successor or facility assignment of a customer in the two solutions is different, the customer is put into the candidate set. Other customers can be added to this subset according to a Bernoulli distribution in which the success probability is determined at the beginning of the ALNS run. There are three greedy repair operators that use different versions of best insertion in which the insertion order of customers is determined by the average accumulated insertion costs from previous iterations. The operators differ in which part of the cost function they consider (location and routing costs, routing and vehicle costs, or location, routing, and vehicle costs). The acceptance is SA-based. If the new solution improves the current best solution, a local search using intra-route relocate and exchange is performed. The ALNS terminates after a given number of iterations without improvement. Then, for a given number of generations, the described ALNS is executed on the starting population to generate additional individuals. Finally, solutions are selected for the next generation based on their quality and their contribution to population diversity. Moreover, facility locations that are not used by any of the individuals are set tabu for the next generation with a higher probability than those that are used in the current generation. The performance of the ALNS on the Barreto, Tuzun, and Prins instance sets is comparable to that of the TBSA of Schneider and Löffler (2019).

Apart from the described algorithms, some heuristics are not competitive over all commonly used benchmark sets or do not report results for all of them, but they perform well for some standard LRP instances. Ferreira and Queiroz (2018) obtain good-quality solutions for the Tuzun instance set, but the results on the Barreto and Prins sets are not competitive. Akpunar and Akpinar (2021) find good-quality solutions for the Barreto set, but their heuristic is dominated by other methods for the Tuzun and Prins sets. X. Yu et al. (2019) identify a large number of new BKSs for the seldomly used Duhamel instances but are not competitive on the Prins set. Results for the Barreto and Tuzun sets are not reported.

2.3 Approximation methods

Gørtz and Nagarajan (2016) study the k-LRP, in which exactly k facilities must be located, and one vehicle is assigned to each facility and must be routed at minimum cost. The authors reduce the k-LRP to the k-median forest problem, a new problem in which k facilities must be located such that the sum of the distance between each customer to its closest opened facility and the length of the minimum spanning tree obtained considering only one facility is minimized. In particular, they prove that if there is an α-approximation algorithm for the k-median forest problem, then there is a 4α-approximation algorithm for the k-LRP. The authors analyze a t-swap LS, i.e., an LS in which, in each iteration, up to t facilities can be swapped. The authors prove that this LS achieves an approximation ratio of $(3 + 2/t)$ for the k-median forest problem. Consequently, the approximation ratio for the t-swap LS for the k-LRP is $(12+\epsilon)$, where $\epsilon = 8/t$. No computational experiments to study the empirical performance of the t-swap LS on k-LRP instances are performed.

Heine et al. (2023) develop a polynomial-time algorithm to approximate the cost of a standard LRP with arbitrary facility capacities, identical vehicle capacities, and no vehicle fixed costs. The algorithm gives worst-case guarantees for both facility capacity violations and costs. It is based on the optimal solution of an MST problem and approximated solutions to a clustering problem, a capacitated facility location problem (CFLP), an assignment problem, and a traveling salesman problem (TSP). Given a parameter $\epsilon \in (0, 1]$, the algorithm computes a solution in which every facility receives at most a load equal to its capacity plus ϵ times (i.e., a small fraction) the vehicle capacity. Because facility capacities are much larger than vehicle capacities for many applications (e.g., regional warehouses in e-commerce or wholesaling), the violation of the facility capacity is relatively small. The solution cost is bounded by $4 + 2\alpha/\epsilon$ times the optimal cost, where α is the approximation guarantee of an algorithm used to compute the lower bound of the CFLP. The current best-known approximation factors for the CFLP are $\alpha = 3$ for the case of identical facility capacities and $\alpha = 5$ for the general case. Four algorithm versions obtained by modifying and/or adding components are empirically tested. The best variant uses LKH—the implementation of Helsgaun 2000 of the S. Lin and Kernighan (1973) heuristic—as a post-optimization phase applied to the routes obtained from the basic version of the approximation algorithm. It returns feasible solutions for all instances in the Barreto, Tuzun, Prins, and Schneider set. The average deviation to the BKSs is 11.69% for the Barreto, Tuzun, Prins sets considered together, and 5.23% for the Schneider set. In addition, it finds 14 new BKSs on the Schneider set.

3 Location-routing problems with pickup and delivery and hub location-routing problems

LRPs with pickup and delivery (LRPPDs) are LRPs in which customers can require both delivery and pickup of goods. Under this common name there lies a plethora of different problems motivated by various practical applications. In this section, we classify these problems based on the origin and destination of transported goods: (i) they flow between facilities and customers (Section 3.1) or (ii) between pairs of customers (Section 3.2). A large class of problems known in the literature as hub LRPs (HLRPs) naturally fits within the second category. Indeed, a contribution of this section is to establish a link between HLRPs and LRPPDs. Table 3 in the online companion summarizes the main characteristics of the surveyed papers on LRPPDs and HLRPs.

3.1 Problems with goods flowing between facilities and customers

Within the class of problems in which goods flow between customers and facilities, we identify two subclasses. In settings where it is inconvenient for the customer to be visited repeatedly, pickup and delivery must occur during the same visit and, consequently, on the same route (Section 3.1.1). For example, customers are the final consumers of the goods, and facilities are urban warehouses or shops. In other settings, pickup and delivery are carried out on separate routes (Section 3.1.2). The reasons are either operational simplicity or that the goods delivered and picked up are so different that they require different vehicles. This setting often arises when customers and facilities belong to the same organization, e.g., customers represent company stores, and facilities are company warehouses.

3.1.1 Pickup and delivery in the same route

Most of the literature considering pickup and delivery in the same route focuses on the case of simultaneous pickup and delivery, i.e., both operations occur during a single visit, usually to increase customer comfort. The only exception is the work of Domínguez-Martín et al. (2024), in which customers can only require pickup or delivery but not both.

From the methodology viewpoint, little attention is dedicated to formulations and exact methods, with few exceptions (e.g., Karimi 2018), and most works present heuristic algorithms. Although a standard version of the LRP with simultaneous pickup and delivery exists (see, e.g., Karaoglan, Altiparmak, et al. 2011), the heuristics all address special cases with additional constraints, or alternative objective functions (Karaoglan and Altiparmak 2015; Leng et al. 2018; X. Wang and Li 2017; V. Yu and S.-Y. Lin 2015b; Zhao, Leng, and C. Zhang 2019; Zhao, Leng, J. Zhang, et al. 2020). Some works also study LRPs with simultaneous pickup and delivery in a multi-echelon setting (Demircan-Yildiz et al. 2016; Y. Wang et al. 2020). In Section A.2 of the online companion we present a formulation for the LRP with simultaneous pickup and delivery.

3.1.2 Separate delivery and pickup routes

If the facility-to-customer and customer-to-facility directions are kept separate, they do not interact, and the fact that some vehicles are used for pickup and others for delivery is irrelevant. In this case, the problems could be addressed using the methodology developed for their delivery-only (or pickup-only) counterparts.

In our opinion, the most important practical setting that requires a distinction between pickup and delivery routes occurs when they are part of a multi-echelon system. For example, consider a two-echelon system in which goods flow from a depot to the customers through facilities and vice versa. The first echelon links the depot with the facilities, and the second echelon links the facilities with the customers. A second-echelon delivery route cannot start before a first-echelon route delivers the goods to the correct facility. Conversely, a first-echelon vehicle cannot depart from a facility before the appropriate second-echelon vehicles have returned with the goods they picked up. Although this synchronization would be required in daily operations, it is not included in the current literature. A possible reason for this simplification is that the required modeling complexity is not justified at the strategic and tactical levels.

Gianessi et al. (2016) study a problem with open routes, and pickup and delivery open routes are structurally different, thus justifying separate treatment. The authors introduce the Ring LRP, a two-echelon LRP in the context of urban last-mile pickup and delivery. Depots located outside the city send/receive goods to/from customers within the city. A decision maker must open facilities to consolidate demand between depots and customers. For simplicity of exposition, we describe the journey of goods delivered from depots to customers; analogous considerations apply for goods to pick up. Goods travel from their origin depot to a facility via direct shipments, and they are delivered to customers through multi-stop vehicle routes. However, the facility receiving the parcel from the depot is not necessarily the same one sending it out for delivery: facilities are connected through a Hamiltonian tour (the "ring") on which goods flow clock- and counterclockwise. The second-echelon vehicle routes also have some peculiarities; most notably, they can end at the origin facility, another facility, or some parking lot (analogously, pickup routes can start at these locations). The authors devise a model with an exponential number of columns (each delivery and each pickup route is associated with one column) and an exponential number of rows (including subtour elimination constraints). Even when improving the model using valid inequalities, the size of the formulation makes it impractical to solve larger instances. Using a matheuristic algorithm, the authors find solutions to instances with up to 80 pickup and 80 delivery customers.

3.2 Problems with goods flowing between customers

Many applications are characterized by transportation requests. In a request, goods travel from an origin, called the pickup customer, to a destination, called the delivery customer. At a small scale (for example, a city courier service), goods can travel directly from origin to destination. At a larger scale (e.g., the national postal service), they must be routed through facilities that act as transshipment points. These facilities consolidate demand and leverage economies of scale in facility-to-facility transportation, usually over large distances.

In the rest of this section, we examine these two cases separately: if direct shipments between customers are allowed or even required (Section 3.2.1), and if goods must flow through facilities (Section 3.2.2).

3.2.1 Direct shipment allowed or required

In this case, the goods picked up from one customer can be directly delivered to another customer without passing through a facility. Such a condition is sometimes allowed and sometimes even imposed. In the latter case, the facilities only act as start and end points of the vehicle routes, but no goods flow through them. As an application area, consider courier services picking up documents and small parcels from a given person and delivering them to another person. We have found one work within the scope of this survey, which belongs to this subclass.

Capelle et al. (2019) study a setting in which direct shipment is required, both pickup and delivery customers have associated time windows, and facilities are uncapacitated. In their extended formulation, they use an exponential number of binary variables, one for each possible route starting and ending at each facility location. They adapt the branch-and-price (B&P) approach of Ropke and Cordeau (2009) for the pickup and delivery problem with time windows. The authors generate new columns by solving a shortest path problem with pickup, delivery, and time windows for each facility with a labeling algorithm. They propose four branching rules; one of them changes the nature of the pricing subproblem and requires introducing an extra label

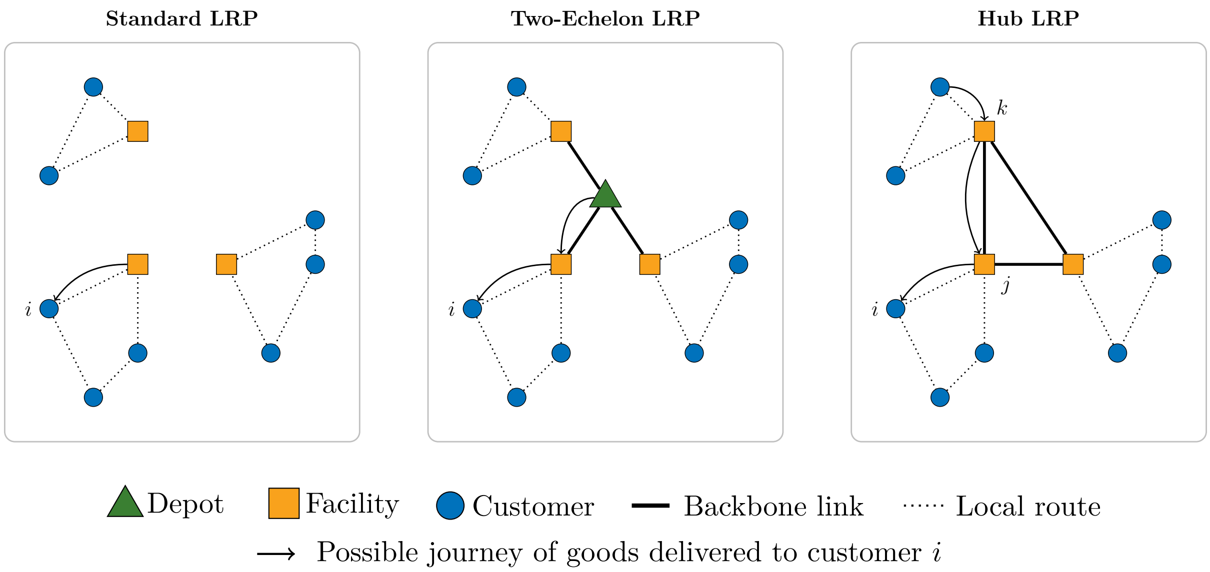

Figure 3: Example networks resulting from the solution of a typical standard LRP, 2ELRP and HLRP.

resource. They also adapt graph preprocessing techniques and pricing heuristics from the literature to speed up the algorithm. Computational tests are carried out on new synthetic instances with clustered customer locations. The proposed method consistently produces solutions with an optimality gap smaller than 0.6% for instances with seven potential facility locations and 30–40 transportation requests.

3.2.2 Goods must flow through facilities

Some LRPPDs forbid direct shipments between customers and instead require that all picked-up goods be first delivered to a facility. From there, the goods are possibly transferred to other facilities, and finally, they are transported to the delivery customer. Because location decisions generally concern long-term strategic planning, it would make little sense to model individual parcels whose existence is usually only known at operational time scales. Indeed, no work in this class models individual pickups and deliveries; rather, goods movements are aggregated in space and time.

Problems of this type are generally known as hub LRP (HLRP), a broad category of LRPs first named as such by Aykin (1995). HLRPs are similar to two-echelon LRPs (2ELRPs; see, e.g., Cuda et al. 2015; Drexl and Schneider 2015). The difference between these two problem classes is that, in HLRPs, there is no central depot. Figure 3 highlights the differences between standard LRPs, 2ELRPs and HLRPs. Note that, in the figure, all HLRP facilities are linked to each other in a so-called complete hub network. In general, this need not be the case (see "Facility-facility routes" below).

In principle, HLRPs can be used in delivery-only contexts. For example, consider a large online retailer keeping its inventory in warehouses and fulfilling orders from the open warehouse closest to a customer. If a customer (i in the figure) orders an item that is not available at their closest facility (j), this item could first be shipped from another facility (e.g., k in the figure). Despite the popularity of online retail, we could not find any work in the recent literature that includes this setting among the possible application areas of HLRPs. Conversely, many authors highlight this problem's importance in designing public transport networks. In this case, smaller stations correspond to customers and large transport hubs to facilities. Flows of goods between customers can be interpreted as the origin-destination demand between pairs of stations. A pickup corresponds to a traveler starting their journey, and the related delivery corresponds to the end of the journey.

HLRPs mainly differ with respect to the following characteristics:

Flow: Some problems consider customer-to-customer flows, i.e., for each pair of customers, they assume that the amount of goods to send from one to the other is known (see, e.g., Nagy and Said Salhi 1998). Other problems do not consider flow explicitly and only ask to open facilities and devise routes that can then be used to send goods between any pair of customers. Examples include the many-to-many HLRP of Saraiva de Camargo et al. (2013), for which we present a model in Section A.3 of the online companion, and the many-to-many p-location Hamiltonian cycle problem (MMpHLP) of M. C. Lopes et al. (2014).

Facility-facility routes: Facilities can be connected to each other with direct links or routes. In the first case, when direct links are used, either all facilities can send goods to all other facilities (a complete hub network), or the planner must decide on the subset of links to enable (an incomplete hub network). Concerning the second case, when routes are used, selecting the edges that will form the facility-facility routes results in an incomplete hub network. Almost all the surveyed literature considers a single Hamiltonian tour connecting the open facilities. An exception is the inter-modal network considered by Yıldız et al. (2021), in which facilities are connected via paths, and each facility-facility link can be part of multiple paths.

Facility-customer routes: Regarding the local routes that connect facilities to customers, we identify three cases. First, each customer must be visited exactly once, either for pickup, or delivery, or both simultaneously. This is the most common case in the literature. In the second case, separate routes are used for pickup and delivery (thus requiring up to two visits). Finally, some works consider the case of split pickup and delivery, i.e., the decision maker decides the number of visits and the pickup and delivery amounts at each visit.

Capacity: Most papers only take into account the capacity of the vehicles operating the local routes that visit customers. Some works, depending on the application field, include facility capacities, which are defined as the maximum flow that can go through a facility.

The HLRP variant that has received the most attention in recent years is the MMpHLP. The seminal work of M. C. Lopes et al. (2014) established a standard version of the problem and introduced a set of benchmark instances. According to the above characteristics, the MMpHLP does not explicitly consider flows, requires a single Hamiltonian tour linking all open facilities, visits each customer once in a local route, and uses capacitated vehicles for the local routes. Moreover, it requires that each open facility is the starting (and ending) point of exactly one route, it fixes the number of facilities to open (therefore the p in the problem name, like, e.g., in the p-median problem), and it does not take into account fixed costs for opening facilities. All reviewed papers propose heuristics (De Freitas et al. 2023; Pandiri and Singh 2021; Ratli et al. 2020) for the MMpHLP and compare the efficiency of the proposed algorithms with at least a selection of previously published approaches. We remark that some of the above authors use different names to refer to the MMpHLP.

In the following, we describe two works that use direct links between open facilities. Ghaffarinasab et al. (2018) consider a continuous approximation of an HLRP motivated by the fact that, when making strategic decisions, the planner lacks detailed operational information, such as the precise locations of customers. The authors assume that each customer is allocated to the nearest open facility, leading to a Voronoi partition of the planning area. Inside each Voronoi cell, the authors estimate the cost of two VRPs (one for pickup and one for delivery) using the approximation formula of Daganzo (1984). They use two heuristic approaches to find the optimal locations of the facilities: one based on Weiszfeld's algorithm (Weiszfeld 1937) for the geometric median, and one using particle swarm optimization.

Wu et al. (2022) propose an ALNS to address an HLRP in which each vehicle can perform split pickups and deliveries. Each customer can be visited by multiple routes departing from different capacitated facilities. In the destroy phase of the ALNS, operators change which facilities are open, which customers are (partially) assigned to which facility, and the local routes. The authors use one greedy and one regret repair operator. Recomputing the objective function during destroy and repair operations is expensive. For example, inserting a customer in a route can change the cost of that route, the flow between facilities, and the cost of other routes. To avoid these expensive computations, the authors work with a simplified objective function. They use an approximation which, given a set of open facilities and routes, does not ensure that the shipped quantities minimize the cost of the inter-facility flow.

Other works assuming direct facility-facility connections include application-specific characteristics. For example, Kartal et al. (2017) and Basirati et al. (2019) require simultaneous pickup and delivery; the latter also considers time windows and split service. Bostel et al. (2015) study an application to postal services using capacitated facilities. Ibnoulouafi et al. (2023) add a sustainability dimension by introducing an emission budget.

4 Location-routing problems with profits

LRPs with profits (LRPPs) arise if not all customers must be served, and the planner must consider the tradeoff between the costs to operate the facilities and route the vehicles and the profits earned by serving a customer. Similar to location decisions, market selection decisions (i.e., deciding which customers to serve) often happen at the strategic or tactical level and influence the operational-level routing. Therefore, including a market selection aspect seems a natural extension of the LRP. Still, only a handful of recent papers consider this setting. By contrast, the literature on VRPs with profits is well-established and includes standardized variants such as the orienteering problem (OP), the prize-collecting traveling salesman problem (PCTSP), the profitable tour problem (PTP) and their multi-vehicle versions. The main difference between these problems is how they model the tradeoff between visiting more customers and spending resources (money or time) to do so. In the OP, the goal is to collect the highest possible profit under a maximum route duration. In the PCTSP, the planner aims to find the shortest route that collects a given profit. In the PTP, the goal is to maximize the difference between collected profit and travel cost.

In LRPPs, the most popular approach is to maximize the difference between profits and costs (as in the PTP), although some authors consider problems in which they maximize profits subject to maximum route duration constraints (as in the OP). We could not find any work that uses the PCTSP approach of minimizing route duration under a minimum profit bound. We describe the PTP-like works first and the OP-like works at the end of this section. We also refer the reader to the recent survey of Dursunoğlu et al. (2025) that places LRPs with profits in the context of selective routing problems, i.e., problems involving a routing component and in which not all customers must be served. Table 4 in the online companion summarizes the main characteristics of the surveyed papers on LRPPs.

The commonly assumed setting in PTP-like problems involves fixed costs for opening facilities and using vehicles, variable routing costs, and profits earned for serving customers. Negrotto and Loiseau (2021) propose a formulation for the LRPP with a homogeneous fleet and symmetric travel costs. This formulation has a polynomial number of variables and an exponential number of vehicle capacity constraints. We present the formulation in Section A.4 of the online companion. The authors add valid inequalities and optimality cuts by adapting existing ones for the standard LRP and the CVRP and by devising new ones specifically for their problem. These inequalities are separated within a branch-and-cut (B&C) algorithm with a custom branching strategy. The authors test the algorithm on a new set of instances specific to their problem and on the Prins instances for the standard LRP. For the latter, they compare their approach with the B&C algorithms of Belenguer et al. (2011) and Contardo, Cordeau, et al. (2013), obtaining slightly larger gaps.

Bagheri Hosseini et al. (2019) study a problem arising in electronic waste collection. The collecting company earns a profit for each appliance they collect. This problem is peculiar in two ways. First, the realized demand of each customer, which represents a geographical area, depends on the incentives that people living in the area receive for recycling their appliances. The incentive amount is a continuous decision variable that influences the demand realization. Second, the collection company can decide to serve only part of the demand; in this case, it pays a partial incentive and collects the corresponding partial profit. The authors develop an ILS algorithm with several problem-specific local search operators.

Ahmadi-Javid et al. (2018) consider a similar problem, the profit maximization LRP (PMLRP), framed in a delivery rather than a pickup setting. In the PMLRP, the authors consider a distribution network in which customers representing geographical areas buy goods from a retailer. Each customer's demand is price-sensitive, and the price is a decision variable. Although the two problems share most of the defining characteristics, there are two main differences. First, the PMLRP does not allow demands to be served partially. Second, price sensitivity is modeled through a discrete distribution that considers a small set of possible prices and their corresponding demands. Ahmadi-Javid et al. (2018) present an extended formulation that they solve with a B&P algorithm. The pricing problem is a new variant of the elementary shortest path problem with resource constraints (ESPPRC). A managerial analysis shows that the proposed model achieves significantly higher profits compared to the price-insensitive case that only considers one price and one corresponding demand and to the sequential case that first decides the facility locations and the prices and then takes routing decisions.

Problems that use the OP approach are often used when the profits are not monetary but represent the importance of visiting a given location. Typically, such a situation arises in military operations where the customers represent targets and the vehicles are attack or surveillance aircraft. Yakıcı (2016), Nadizadeh (2021), and Yılmaz et al. (2019) present mathematical formulations and heuristic approaches for the resulting orienteering LRP.

5 Location-routing problems with time windows

LRPs with time windows (LRPTWs) are defined as LRPs in which each customer requires to be served within a specific time interval (for a mathematical formulation, see Appendix A.5 in the online companion). Applications include e-commerce deliveries, in which customers can often express preferences for when they want to be served, or deliveries for specific industries (e.g., cash-in-transit or industrial waste collection), in which workers (i.e., the customers) are only available to load or unload goods at specific times. Table 5 in the online companion summarizes the main characteristics of the surveyed papers on LRPTWs.

Ponboon et al. (2016) are the first to propose an exact method for the standard LRPTW. They design a B&P algorithm based on a set partitioning formulation using an exponential number of route variables. The authors decompose the pricing subproblem by solving, for each candidate facility, an ESPPRC in which resources ensure that capacity and time window constraints are respected. The branch-and-bound (B&B) tree is explored depth-first using branching rules operating on the facility variables, the number of vehicles per facility, and the arc variables. The authors improve the B&P convergence using the dual perturbation method of Du Merle et al. (1999). Preliminary results showed that the B&P algorithm could only solve instances with up to 40 customers within acceptable runtimes. Hence, the B&P algorithm is validated on a subset of the Barreto instances having a maximum of 36 customers and five facilities to which time windows of unrestricted length are added so that the LRPTW optimal solutions are those optimal for the standard LRP. Moreover, the authors introduce the new Ponboon instance set that includes the first 10, 25, and 40 customers of the R1 Solomon VRP with time windows (VRPTW) instances, and three facility locations generated through a clustering technique. These instances are solved to optimality, with reported runtimes ranging from a few seconds for instances with ten customers to nearly eight hours for those with 40 customers.

Farham et al. (2018) propose a B&P algorithm similar to the one of Ponboon et al. (2016). However, the authors use additional well-known acceleration strategies (initially proposed for the VRPTW by Baldacci, Mingozzi, and Roberti, 2011 and Contardo, Desaulniers, et al., 2015). The authors also propose a two-stage heuristic. In the first stage, a CFLP is solved in which the solution consists of direct deliveries from the chosen facilities to each customer respecting their time windows and facility capacities. In the second stage, the opened facilities are fixed in the master problem used by the B&P algorithm, and routes are obtained by solving an MDVRP with time windows (MDVRPTW) with a simplified and faster version of the proposed B&P algorithm. On a new set of LRPTW instances, good-quality solutions are obtained quickly by the two-stage heuristic. The results suggest that the upper bound provided by this heuristic allows the exact B&P algorithm to run faster. The authors also solve the Ponboon instances with their B&P algorithm. However, we refrain from comparing the performance of the algorithms of Ponboon et al. (2016) and Farham et al. (2018) because different solutions are reported as optimal in the two papers, as already noted in Farham et al. (2018). The reason for this difference in the solutions remains unclear. The described method is the only heuristic for the standard LRPTW proposed within the time frame of our survey.

In the following, we discuss LRPTW variants. Koç, Bektaş, et al. (2016a) study the fleet size and mix LRPTW (FSMLRPTW), which extends the LRPTW by considering a fleet of vehicles with different capacities and fixed costs. The authors propose a MIP. Different variable aggregation and constraint disaggregation strategies produce four additional formulations. The authors also adapt four well-known polynomial-size valid inequalities to the FSMLRPTW. To heuristically solve the problem, they propose a hybrid of GA and ALNS. They first generate one starting solution by applying a commonly used procedure for assigning customers to facilities. For each facility, routes are built by applying the algorithm of Clarke and Wright (1964) using the capacity of the largest vehicle type for all routes and considering time windows. Randomized destroy and repair operators are applied to the obtained solution to fill the initial population. The authors use the parent selection and crossover procedure of Koç, Bektaş, et al. (2015), and a new greedy procedure splits the resulting chromosomes into vehicle routes. They also adapt the education phase of Koç, Bektaş, et al. (2015) to consider the possibility of closing or opening a facility. The new generation is added to the existing population, and a fraction of the elite solutions undergo the intensification procedure of Koç, Bektaş, et al. (2015). Finally, a mutation procedure is applied to a random non-elite solution. The algorithm stops upon reaching a given number of iterations without improvement. The authors generate a new instance set based on the one developed by F.-H. Liu and Shen (1999) for the fleet size and mix VRPTW. Regarding the mathematical formulation, the results show that variable aggregation and the four valid inequalities decrease CPLEX optimality gaps and runtimes. When compared to the solutions obtained by solving the best mathematical formulation with CPLEX, the best run of their algorithm always obtains gaps below 0.05% (averaged by instance subset) for the smaller instances, and it yields better solutions for the larger instances.

Çetinkaya et al. (2018) consider an LRP with an unlimited vehicle fleet and time windows on arcs. Fixed vehicle costs are neglected. The problem is motivated by areas affected by terrorism, in which some roads are only safe during the daytime, or by areas subject to traffic regulations that forbid the entrance of trucks into the city center at some hours of the day. Rave and Fontaine (2025) propose an ALNS for an LRPTW with load-dependent travel times motivated by the use of cargo bikes for last-mile delivery. In this setting, the load on a bike and the street gradient significantly change its speed. Managerial insights from realistic instances confirm that including these aspects results in opening more facilities at higher elevations and assigning customers to facilities with similar altitudes.

6 Latency location-routing problems

Latency LRPs (LLRPs) assume that all customers prefer to be visited as early as possible (for a mathematical formulation, see Appendix A.6 in the online companion). To this end, LLRPs minimize the sum of the arrival times at the customers (called the total arrival time from now on). LLRP applications include supply chains for perishable products and supplies distribution to affected areas in post-disaster relief activities. In the latter case, it is crucial to minimize the waiting time of the recipients because deaths and losses increase as time passes after the disaster. In the standard LLRP, at most a given number of uncapacitated facilities can be opened. There is always an optimal solution in which the maximum number of available facilities is opened because no facility opening costs are included in the objective function, and an open facility is not required to serve any customers. Moreover, if the matrix of the travel times satisfies the triangle inequality, the number of used vehicles in an optimal solution is the minimum of the number of customers and the number of available vehicles because the cost of the arc connecting the last customer of each route to the facility is not part of the objective (Osorio-Mora, Rey, et al. 2023). Table 6 in the online companion summarizes the main characteristics of the surveyed papers on LLRPs.

Moshref-Javadi and Lee (2016) formulate a MIP and propose two lower-bounding and one upper-bounding procedure as well as a memetic algorithm (MA) for the standard LLRP. In the numerical experiments, the authors interpret the standard LRP benchmark sets Tuzun, Barreto, and Prins as LLRP instances and compare the results of their MA to the derived lower and upper bounds. In addition, they show the importance of considering latency as objective function in customer-oriented distribution systems and disaster relief as follows: first, they compute the total latency of the standard LRP solutions of Vincent et al. (2010) on the Tuzun instances. Then, they solve the Tuzun set interpreted as LLRP instances with the same number of opened facilities and used vehicles as in the solutions of Vincent et al. (2010) and compare the solutions.

Nucamendi-Guillén, Martínez-Salazar, et al. (2022) propose two formulations for the standard LLRP that are based on multi-level networks. The authors also extend these formulations to the LLRP with opening costs (LLRPOC), which minimizes a weighted sum of the total arrival time and the facility opening costs. Their network (similar to the one for the cumulative CVRP of Nucamendi-Guillén, Angel-Bello, et al. 2018) is composed of as many levels as the maximum number of customers in a route (computed by considering demands and vehicle capacity). Each level contains a copy of each customer and represents the backward position of a node in a route (i.e., the first level represents the last position). In all levels except the first, each customer is connected to all other customers of the following level via an arc. Moreover, all levels except the first include a copy of the candidate facilities. The two formulations differ in how they represent the connection from the facilities to the customers. In the first formulation, each potential facility is connected to a dummy node that, in turn, is connected to every customer of the next level. In the second formulation, potential facilities are directly linked to customers. Because both networks are acyclic, connectivity can be ensured using a polynomial number of connectivity constraints. The authors propose three exact algorithms—a B&C, and two variants of a branch-and-check (Thorsteinsson 2001)—and a hybrid of GRASP and ILS as heuristic solution method. The performance of the models and algorithms is tested on the Barreto and Prins sets interpreted as standard LLRP instances. For small instances, the formulations can be directly solved with Gurobi without requiring the B&C or branch-and-check algorithms. The results do not show evidence of the superiority of one formulation over the other, but both formulations outperform the one proposed by Moshref-Javadi and Lee (2016). Among the three exact methods, the B&C algorithm is the slowest but finds the largest number of feasible and optimal solutions. The GRASP/ILS outperforms the MA of Moshref-Javadi and Lee (2016) with respect to solution quality; runtimes are not compared. On 20 newly generated LLRPOC instances, the GRASP/ILS returns the same solution obtained by the best of the exact methods (run with a time limit) for 14 of the instances and a better solution for one instance.

Osorio-Mora, Rey, et al. (2023) propose a hybrid of SA and VND for solving the standard LLRP. The authors apply the k-means clustering algorithm (Macqueen 1967), the LKH-3 heuristic for the cumulative CVRP (Helsgaun 2017), and a repair procedure to generate a feasible starting solution. Then, as long as the minimum temperature or a given number of iterations without improvement is not reached, the current solution undergoes a diversification and an intensification phase. In the diversification phase, the authors apply inter- and intra-route moves. Other operators modify the set of open facilities and the number of vehicles allocated to each facility. In the intensification phase, the resulting solution is improved by a VND procedure. Finally, the best-found solution is further improved via the LKH-3 heuristic. The authors compare their SA/VND algorithm to the methods of Moshref-Javadi and Lee (2016) and Nucamendi-Guillén, Martínez-Salazar, et al. (2022) on the TUZUN, BARRETO, and PRINS interpreted as LLRP instances. The results show that SA/VND is more robust than the competitors because the difference in solution quality between the best and the average run is small. The solution quality of SA/VND is also superior to the competitors, but runtimes are higher. However, SA/VND requires less time to reach the same solution quality as the competing methods.

Osorio-Mora, Escobar, et al. (2023) propose a hybrid of ILS, SA, and VND for solving the standard LLRP and the multi-depot cumulative CVRP. To generate initial routes, the LKH-3 heuristic first solves a TSP to obtain a giant tour visiting all customers. A clustering procedure is applied to split the giant tour into groups of consecutive customers, such that the load of each cluster does not exceed the vehicle capacity. For each facility-cluster pair, the authors compute the allocation cost as the total latency of the route composed of the respective facility and the customers in the cluster. Finally, an integer linear program produces the best facility-cluster assignment. The ILS uses three perturbation procedures of different strength that are applied with different probabilities. During the ILS, infeasible solutions with regard to vehicle capacities are allowed and handled by means of a penalty mechanism. In each iteration of the ILS, a pure LS based on five neighborhood operators is executed. Then, a slightly simplified version of the SA/VND of Osorio-Mora, Rey, et al. (2023) is run to further improve the quality of the solution. When the ILS terminates, for each opened facility in the best found solution, a cumulative CVRP is solved by applying the LKH-3 heuristic, and the LS procedure described above is executed again. Finally, for each route, if the first visited customer is not assigned to its closest facility, the route is assigned to the closest facility. Compared to Moshref-Javadi and Lee (2016) and Osorio-Mora, Rey, et al. (2023), the algorithm dominates on the TUZUN instance set, and it is competitive on the PRINS and BARRETO sets (all interpreted as standard LLRP instances).

Zou et al. (2024) propose an MA enhanced by a reinforcement learning component for the standard LLRP. To generate the initial population, the authors first randomly open the maximum number of facilities allowed in the respective instance. Then, routes are constructed using either a greedy or a random method selected with equal probability. Three parents are randomly selected at each iteration, and a multi-parent edge assembly crossover generates an offspring solution. This crossover operator modifies the TSP operator of Nagata and Kobayashi (2013) to account for the presence of facilities and multiple directed routes. The resulting offspring is repaired if infeasible and mutated with a given probability. Then, a VND is applied, which allows infeasible solutions, and Q-learning determines the exploration order of the neighborhoods. The obtained offspring is added to the population, the fitness (based on quality and diversity) of all solutions is reevaluated, and the solution with the worst fitness value is removed from the population. If the best solution remains unchanged for a given number of iterations, 50% of the population (except for the best solution) is randomly removed. New solutions are added either with the procedure used to generate the initial population or by randomly selecting the oldest solutions from a limited memory storing the local optima found in the VND. The results of the computational experiments performed on the TUZUN, BARRETO, and PRINS sets interpreted as

LLRP instances show that their algorithm outperforms all heuristics from the literature with regard to solution quality and speed. Moreover, the results confirm the benefits of using a multi-parent edge assembly crossover operator with three parents, as opposed to using the same operator with two parents or a simpler order crossover operator. The authors also show the advantages of using Q-learning for determining the neighborhood exploration order by comparing the results of their algorithm with two variants: one that explores neighborhoods in a random sequence and another that follows a fixed order based on increasing neighborhood complexity.

7 Other location-routing problem variants

In this section, we review LRP papers whose distinguishing features are not covered by any of the defining attributes of the previous sections, but the number of collected papers does not justify separate sections. For some of them, we identified high-level characteristics shared among groups of papers that allow us to divide them into general problem classes. To enhance readability and highlight the similarities of these papers, we group them into: LRPs with time dependence (Section 7.1), LRPs with environmental considerations (Section 7.2), LRPs with non-standard location and routing decisions (Section 7.3), cooperative and competitive LRPs (Section 7.4), location arc routing problems (LARPs, Section 7.5), and miscellaneous (Section 7.6).

7.1 Location-routing problems with time dependence

In real-world applications, travel times are typically not constant and change dynamically throughout the day, for example, due to traffic volumes. Because travel time patterns are likely to repeat on a periodic basis, it makes sense to integrate time-dependencies into strategic problems like the LRP. Schmidt et al. (2019) focus on time-dependent travel times and introduce the time-dependent LRP that integrates the LRP with the time-dependent VRP (TDVRP, Malandraki and Daskin 1992). In their problem, exactly one facility must be opened, the number of vehicles at each potential facility is restricted, and no fixed costs for opening a facility or using a vehicle are considered. In the proposed mathematical formulation, the authors consider a limited time horizon discretized into intervals of equal length, and they assign a constant travel time to each arc-interval pair. The authors introduce preprocessing techniques and valid inequalities to strengthen the model formulation. They also propose a heuristic algorithm, in which they fill a solution pool using different construction methods, eliminate dominated routes from the pool, and finally solve a set partitioning model on the remaining routes using Gurobi. Their newly introduced instance set is based on the road network and traffic of Quebec City and has up to five facilities, 100 customers, and 15 time intervals. The computational results show that the valid inequalities improve the lower bounds, and that the matheuristic generates better-quality solutions faster compared to Gurobi with a three-hour time limit.

For some applications, like, e.g., planning of public bus networks, it can also be reasonable to integrate time-dependent demands into an LRP setting. V. F. Yu et al. (2021) study the LRP with time-dependent demand (LRPTDD) with capacitated facilities and time windows. In the LRPTDD, customers produce quantities of an item at known rates (expressed as production quantity over time unit). These quantities must be picked up, and because they accumulate over time, the picked load depends on the arrival time of a vehicle at the customer. Because the pickup orders of all accumulated customers must be satisfied, certain customers are visited more than once by different vehicles. To solve the problem, the authors propose an SA algorithm that can also be used to solve the standard LRP. The computational results on the benchmark sets Tuzun, Barreto, and Prins show that the SA is dominated by the state-of-the-art algorithms for the standard LRP from the literature. However, on the newly generated LRPTDD instance set, the SA matches the solution obtained by CPLEX with a 12-hour time limit for all instances except one, for which the SA improves the solution quality.

7.2 Location-routing problems with environmental considerations

Some LRPs involve limiting the environmental impact of operations, usually by minimizing CO_2 emissions or fuel consumption, often in some combination with the classical cost minimization objective. Examples often arise in fast-moving consumer goods and cargo shipping industries. Companies such as Unilever, UPS, FedEx, and DHL have decreased their fuel consumption and emissions by modifying their network structures, i.e., the number and location of opened facilities (Dukkanci et al. 2019).

Reducing emissions is vital in population-dense areas, which makes LRPs with environmental considerations especially relevant for city logistics. For example, Koç, Bektaş, et al. (2016b) present a MIP and an ALNS for an LRP with heterogeneous vehicles in which the objective is to minimize the facility opening cost, the fleet composition cost, and fuel and CO_2 emission costs based on the direct fuel costs and the resulting CO_2 emissions as estimated by Barth and Boriboonsomsin (2009). The authors model the city as a grid with customers and facilities on the vertices. The grid is divided into zones with different speeds. The arc traveling speed (which depends on the zone), the vehicle load, and the engine type determine the fuel consumption. In such a context, the shortest path between two vertices is not necessarily the cheapest or the least polluting one because speed and vehicle load affect fuel consumption. The authors heuristically build sets of good-quality paths between each pair of vertices; however, the cost of each path is load-dependent and can only be estimated before the path is part of a complete solution. The ALNS works with these estimated costs and only computes the real cost of a solution periodically. The computational experiments are based on a newly generated instance set. The results show that using the approximated costs in the move evaluations deteriorates the solution quality by only 1% while reducing runtime by approximately 29%. Moreover, the ALNS returns solutions of the same quality as those obtained by CPLEX.

Koç (2018) also proposes a MIP and an ALNS for an LRP with an unlimited homogeneous vehicle fleet and customer time windows. The objective is to minimize the facility opening cost, driver costs (based on route duration), and fuel and CO_2 emission costs calculated as in Koç, Bektaş, et al. (2016b). In addition to typical LRP decisions, the problem demands selecting a speed for each traversed arc from a discrete set of speeds. Dukkanci et al. (2019) consider a similar problem, in which they do not consider driver costs but assume a limited fleet.

S. Wang et al. (2025) present an exact algorithm for an LRP with a heterogeneous fleet of capacitated vehicles, minimizing fixed facility and vehicle costs, routing costs, and CO_2 emission costs. Unlike the other papers in this section, the authors consider a simplified carbon emission model compared to the commonly used model by Barth and Boriboonsomsin (2009). The authors present an extended formulation using route variables, strengthen it with valid inequalities, and solve it via a BCP algorithm in which the pricing subproblem is solved with a bidirectional labeling algorithm. The algorithm finds optimal solutions for instances with up to 100 customers and five potential locations on a new set of instances derived from the Prins and Akca sets. Furthermore, the BCP is competitive with the algorithms of Baldacci, Mingozzi, and Wolfler Calvo (2011) and Contardo, Cordeau, et al. (2014) for the standard LRP.

7.3 Location-routing problems with non-standard location and routing decisions

Some LRP variants require additional or different decisions on the location and/or routing components of the problem. This is the case for the location and location-routing problem (L&LRP), the location or routing problem (LoRP), and LRP variants in which a route between facilities must be determined.

In the L&LRP, the optimal customer locations must be determined in addition to those of the facilities. This variant arises in humanitarian logistics and disaster management; facilities are service centers of a humanitarian institution, and customers are refugee camps. Arslan

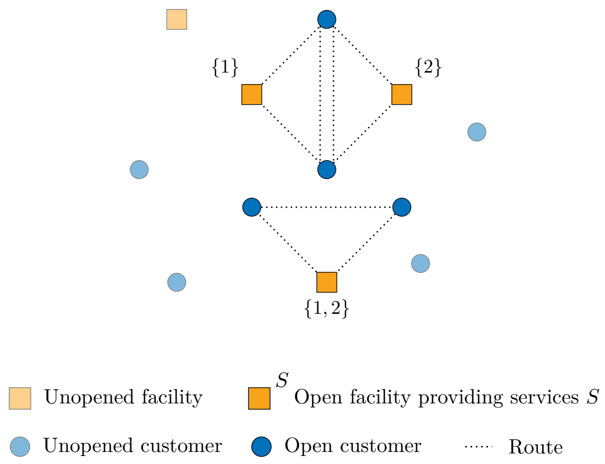

Figure 4: Example solution of an L&LRP.

et al. (2021) introduced the L&LRP motivated by a real-world application with the Turkish Red Crescent. In this problem, capacitated refugee camps must be chosen from a candidate set to host a given number of refugees. A set of potential locations for capacitated service-provider institutions is given. Each institution offers the refugee camps one or more services (e.g., healthcare and education) through routes starting and ending at the institution. Multiple providers can serve a given camp to ensure it receives all services. The objective is to minimize the camp opening and routing costs. Figure 4 depicts the solution of an L&LRP. The authors propose a path-based model and a BCP algorithm using a recursive algorithm to eliminate cycles in the pricing problem, tailored branching rules, and knapsack inequalities to strengthen the LP relaxation. The BCP algorithm is investigated on a set of new instances with up to 244 nodes (representing the number of candidate refugee camps and the number of hospitals, high schools, and municipality city halls) derived from the case study.

In the LoRP, not all customers must be visited by vehicle routes. Customers who are within a given distance from an open facility (called coverage range) are assumed to be covered even if no route visits them. An example solution of an LoRP is shown in Figure 5. An exemplary application of the LoRP is locating schools: if students live close to a school, they can reach it on foot; otherwise, they must be transported by a school bus. Other applications are the location of shopping malls or testing centers during pandemics. The LoRP corresponds to the standard LRP (see Section 2) if the coverage range is zero and to the set covering problem if routing costs are infinite. Arslan (2021) introduce the standard LoRP, which assumes capacitated facilities, an unlimited fleet of capacitated vehicles, and maximum route duration constraints. In addition to classical facility opening and vehicle routing costs, the author considers a cost associated with customers not visited by vehicles. The farther such a customer is from their nearest open facility, the higher the cost. Arslan (2021) proposes a set partitioning formulation solved with a B&P algorithm. Computational experiments on a set of instances derived from the Akca and Prins sets show that the B&P gives average optimality gaps under 3% on instances with up to 200 customers within three hours. A theoretical analysis backed up by computational experiments on random instances with uniformly distributed customers shows that the total cost decreases linearly with the coverage range.

Haghi et al. (2023) study a variant of the standard LoRP with uncapacitated facilities and a limited fleet of capacitated vehicles, in which each customer location contains several individual customers. Locations within the coverage range can be served in full or in part, depending on their distance from the closest open facility (the closer the location, the more individual customers are served). Locations not fully served can be visited by vehicle routes that serve a subset of the customers, the remaining customers (if there are any) are left unserved. The

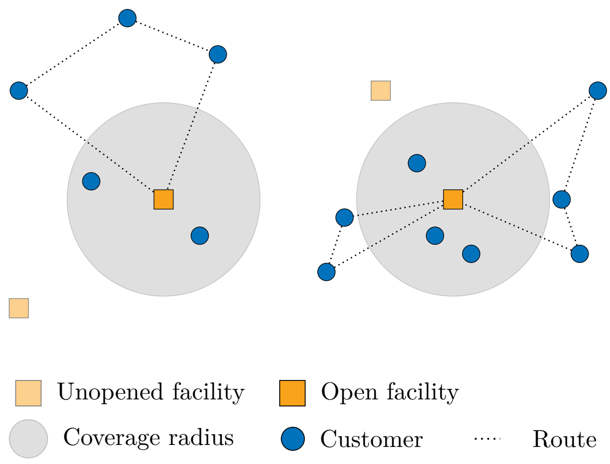

Figure 5: Example solution of an LoRP.

routes must adhere to maximum duration constraints. The authors consider a hierarchical objective function that first maximizes the number of customers served and then minimizes the sum of facility opening costs, vehicle fixed costs, and routing costs. They present an ALNS and two mathematical formulations, a compact one with Miller-Tucker-Zemlin (MTZ) constraints solved with CPLEX and an extended one with subtour elimination constraints solved with a B&C algorithm. On a set of randomly generated small-size instances, the extended formulation provides better dual bounds, while the compact formulation obtains better primal solutions with a time limit of three hours; both formulations find a similar number of optimal solutions. The ALNS matches the best solution found by any of the two formulations for 90% of the instances with a runtime of less than a second. The ALNS is also used to solve a subset of the instances derived by Arslan 2021 and gives solutions with an average gap of 0.47% to the solutions of Arslan (2021), the latter of which are optimal for 60% of the instances; it also finds two new BKSs.

In the problem of Rahim and Sepil (2014), a glass recycling company has to determine the locations of a set of bottle banks to which residents bring glass. The revenue of a bottle bank depends on its proximity to the residents because having a nearby bank increases the willingness to recycle bottles. The company must also determine a single route for an uncapacitated vehicle visiting all opened bottle banks and additional customers (such as hospitals, restaurants, bars, and schools). The objective is to maximize the revenue generated by the glass banks minus the routing cost. Kemmar et al. (2025) present a variant of the standard LRP in which a subset of customers to be determined is served twice instead of once. These customers can act as alternative facilities in case of disruptions to the main network. The case study is based on humanitarian relief to refugees in Lebanon.

7.4 Cooperative and competitive location-routing problems

Cooperative LRPs arise when multiple companies share facilities, vehicles, or both. Conversely, in competitive LRPs, multiple companies compete over a set of customers that will then be served by the company whose facility is closest to them.

Quintero-Araujo et al. (2017) explore the impact of cooperation in a setting in which each company has a capacitated facility, a homogeneous fleet of capacitated vehicles, and a set of assigned customers. The customer sets are disjoint. The authors consider three levels of cooperation: non-cooperative, semi-cooperative, and fully cooperative. In the non-cooperative scenario, each company serves its customers from its facility by solving a VRP. In the semi-cooperative scenario, each company still operates its facility and fleet, but customers of any company can be served from any facility. A solution for the semi-cooperative scenario is obtained by solving

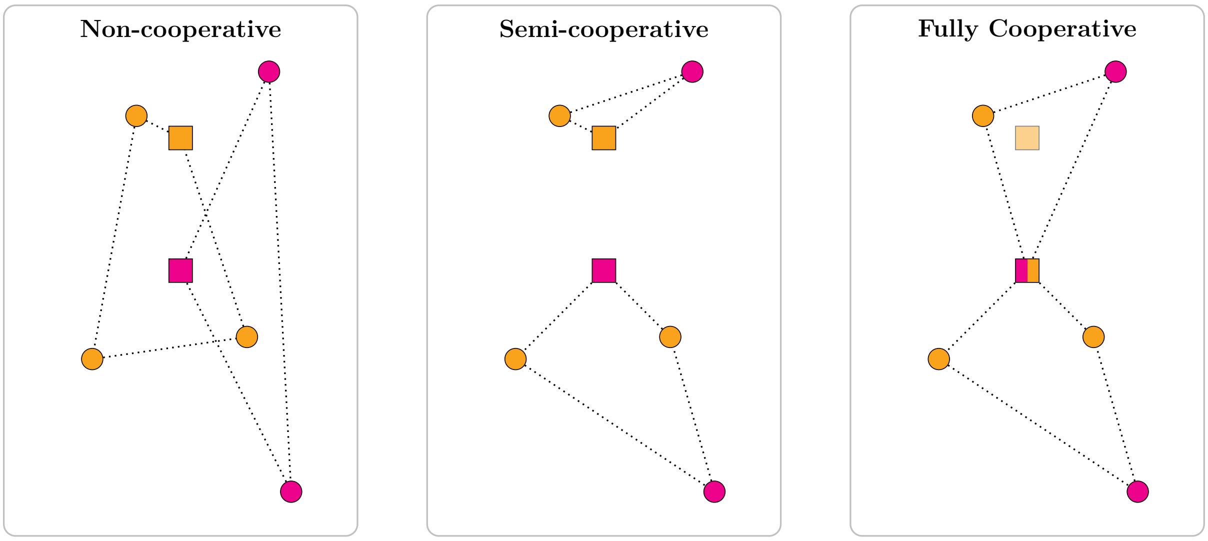

Figure 6: Non-cooperative, semi-cooperative and fully cooperative scenarios in Quintero-Araujo et al. (2017). Squares are facilities, and circles are customers. Different companies correspond to different colors. In the fully cooperative scenario, the semi-transparent square represents a facility location where no facility was opened.

an MDVRP. Finally, in the fully cooperative scenario, companies share facilities and vehicles. In this case, not all facilities need to be opened because operating fewer facilities than there are companies might provide a better solution. A solution for the fully cooperative scenario is obtained by solving a standard LRP (see Section 2) in which the companies' facilities are merely potential facility locations. Figure 6 depicts the difference between the three scenarios. The authors propose a heuristic for each of the three scenarios and perform computational experiments on the Prins, Barreto, and Akca instance sets. As expected, the fully cooperative scenario leads to the lowest costs. However, when computing the CO_2 emissions (which are not part of the objective function), the semi-cooperative scenario yields the lowest values for most instances because it results in more open facilities and shorter routes.

Osicka et al. (2019) introduce the standard location-routing game, in which companies have associated disjoint sets of customers and a fleet of capacitated vehicles. Any subset of the companies can form a coalition; members of the coalition share facilities and vehicles. The cost associated with a coalition is the optimal cost of the corresponding standard LRP. The authors extend this game considering capacitated facilities or upper bounds on the number of facilities each coalition can open. In these extensions, the decisions made before cooperating are not necessarily feasible in their cooperative version. The authors prove that for the standard location-routing game and the variant with a limited number of facilities, there is no incentive for players to deviate from the coalition when the facility opening costs are significantly larger than the routing and vehicle costs.

7.5 Location arc routing problems

LARPs combine location and routing decisions in settings where the service is defined on the links instead of the nodes of the graph. Typical applications are, e.g., in newspaper delivery, garbage collection, road gritting, snow removal, and meter reading.

Fernández, Laporte, et al. (2019) study six variants of LARPs on undirected graphs. The six variants are derived from three base problems by considering capacitated or uncapacitated facilities for each of them. The first two base problems limit the maximum number of facilities to open; the first minimizes the routing costs, and the second the makespan. The third base problem does not limit the number of facilities and minimizes the sum of opening and routing costs in the

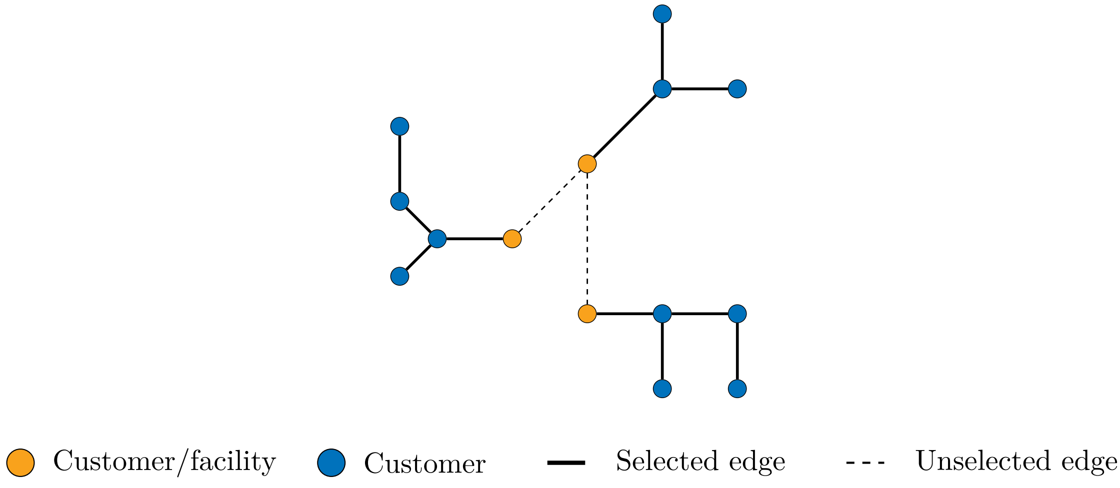

Figure 7: Example solution of a p-LRP.

objective function. The authors derive optimality conditions for each of the six variants, propose two and three-index variable formulations, perform a polyhedral study, and solve the models via a B&C algorithm. Experiments on adapted instances of the multi-depot rural postman problem show that for the two uncapacitated models minimizing costs, the B&C algorithm solves all instances with up to 200 vertices and approximately one third of the instances with up to 744 vertices to optimality. Moreover, the results indicate that the models minimizing the makespan are harder to solve than those minimizing costs and that models with capacitated facilities are more challenging than their uncapacitated counterparts. Finally, the two-index formulation is superior to the three-index formulation in terms of speed and number of instances solved to optimality within the time limit.

7.6 Miscellaneous location-routing problems

In this section, papers on rarely-studied LRP variants. For some LRP applications, routes must follow a special structure. This is the case for the LRP defined on trees, and the LRP on a Riemannian manifold surface. LRPs defined on trees have applications in the telecommunication industry. Aráoz et al. (2019) study several variants of LRPs defined on an undirected tree with no facility or vehicle capacities, for which all optimal routes are bipaths, i.e., the selected edges are traversed exactly twice. The authors first introduce the general p-LRP defined on a tree. In this problem, a facility can be opened at any customer location, p facilities must be opened, and the routing and the facility opening costs must be minimized. Both vertices and edges can have demand. Figure 7 shows the solution of a p-LRP. Despite the problem being defined on an undirected tree, the authors first reformulate it using a directed tree. They devise an integer formulation that has the integrality property, thus proving that the problem is solvable in polynomial time. They also propose optimal greedy polynomial-time algorithms for three p-LRP variants without facility opening costs: the p-vertex-LRP (p-VLRP), in which the demand is located only at the vertices, the p-edge-LRP (p-ELRP), in which the demand is located only at the edges, and the general case, in which both vertices and edges can have demand. Moreover, optimality conditions for the p-VLRP and the p-ELRP with facility opening costs are derived. Finally, the authors study a p-LRP variant without facility opening costs in which not all vertices must necessarily be visited, or the graph induced by demand edges does not span the complete vertex set. To optimally solve this LRP variant, the authors propose a dynamic programming algorithm with a quadratic runtime.

In scenarios with an irregular ground surface, a curved spatial environment must be considered instead of a flat Euclidean plane. To capture the change in the curvature of routes due to the presence of hills and mountains, Tokgöz, Alwazzi, et al. (2014) and Tokgöz and Trafalis (2015) study continuous LRPs on a Riemannian manifold with uncapacitated facilities and geodesic distances.

Some LRP variants arise because of the nature of the transported goods. This is the case with LRPs with special vehicle loading features, LRPs with risk consideration, and LRPs with perishable products. In some food delivery applications, groceries are transported using vehicles with multiple compartments, each exclusive to a specific type of food. For example, dedicated compartments are necessary when transporting frozen and unfrozen food, or multiple compartments are needed to keep contaminating food isolated (Moon et al. 2020). A similar scenario can be found in waste collection, where vehicles have multiple compartments to keep waste separated by type (Shang, Ma, and Y. Liu 2023). In some cases, items cannot be stacked because of their fragility or shape, and multi-dimensional loading constraints must be considered in LRPs (Ferreira and Queiroz 2022). Integrating such operational decisions with the strategic decision of locating facilities is motivated by disaster and military applications, in which the time horizon for making strategic, tactical, and operational decisions is short. One example is the transportation of patients on stretchers after natural disasters (Ferreira and Queiroz 2022). The risk of robbery during routing activities must be considered for applications in which high-value products are delivered from facilities to customers (e.g., cash from banks to automated teller machines). In this context, Allahyari et al. (2021) study the secure pickup and delivery LRPTW (see Sections 3 and 5). The authors consider a history of previously used routes and must plan new routes for a given day. The objective function is a weighted sum of the LRP costs and the planning risk. This planning risk is the sum of the risks of the individual routes; the risk of each route is the maximum risk associated with a customer visited along the route. The customer risk, in turn, is a function of the amount carried, the route similarity to historical routes (both in terms of arrival time and used arcs), and the absolute arrival time. Macedo et al. (2015) study a multi-trip LRP, i.e., more than a single route for each vehicle in the planning period is allowed. The problem is motivated by the transportation of perishable products with tight constraints on the maximum duration of the vehicle routes. The authors formulate a three-index commodity flow and a network flow model and propose a skewed VNS to solve this problem. Computational experiments on a new instance set show that the network flow formulation is stronger than the three-index commodity flow model for small instances. However, the network flow formulation uses a large number of variables (one for each feasible route) and is impractical for larger instances.

V. Yu, Aloina, et al. (2022) study an LRP in which a minimum number of facilities must be open for predefined subsets of facility locations. Motivated by the different prices charged in different areas in the cement and fuel industries, Setak et al. (2018) and Dastaki et al. (2020) study the multi-zone LRP with pricing, in which the company has to decide the optimal price of the products for each zone. The chosen prices affect the customer demand. Tordecilla et al. (2022) study an LRP in which facility capacities must be chosen from a discrete set, as is common when facilities are lockers. The authors propose three MIP formulations and an ILS algorithm. Comparing their solutions to standard LRP solutions of a previous paper by a subset of the same authors (Quintero-Araujo et al. 2017) on Akca, Barreto, and Prins instances shows that moderate cost savings are achieved by the additional sizing flexibility. Shang, Ma, Y. Liu, and Sun (2022) study an LRP in which customers are visited by multiple vehicles, and vehicles must queue if a customer is already being served when they arrive. The authors include the sum of vehicle queuing time in their objective function.

8 Key observations and recommendations

In this section, we first outline our main takeaways from studying the recent LRP literature and then provide specific conclusions and suggestions for future research on the standard LRP and on LRPs with pickup and delivery. Some of these remarks were already included in similar form in the conclusions of the surveys by Drexl and Schneider (2015), Prodhon and Prins (2014), and Schneider and Drexl (2017), however, our analysis of the recent literature suggests that it is reasonable to repeat and extend them.

LRPs simultaneously optimize strategic (location) and operational (routing) decisions. Said Salhi and Nagy (1999) showed that ignoring that transportation is performed using vehicle routes in the location problem and instead assuming direct transports leads to strongly sub-optimal solutions. Jointly optimizing location and routing decisions usually involves simultaneously taking decisions with a different time horizon. In some applications, however, the location decisions are short-term (for example, if the locations are mobile distribution centers, shared warehouses, emergency facilities, pop-up stores, or mobile healthcare providers) or the routing decisions are mid- to long-term (if the customer set is stable and rarely changes such as in waste collection). In these cases, location and routing costs are directly comparable. In all other applications, which constitute the majority of cases, the different time horizons coexist, and strategic and operational costs mix in the LRP objective function. In this case, jointly optimizing location and routing decisions can still bring benefits, as demonstrated by Said Salhi and Nagy (1999). However, it is crucial that the harmonization of strategic and operational decisions is carefully considered, e.g., employing scenarios to model customer demands or creating customer sets and demands that represent long-term trends using a data-driven approach. Authors modeling a real-life problem should be conscious of these aspects and clearly identify them to justify using an LRP-based approach for their problem. We note that this point has been mostly ignored in recent literature.

For most variants studied in this survey, the problems belonging to each variant tend to be diverse, as are the real-life applications motivating them. Therefore, it is absolutely necessary to accurately describe the assumptions and characteristics of the newly introduced variants and clarify the differences and commonalities with existing variants. Papers presenting new variants should provide a precise verbal description of the problem without relying on the mathematical formulation only. At the same time, authors must ensure that the mathematical models are correct and complete for the problem they tackle. A significant number of papers that we reviewed fall short of these expectations. Therefore, we encourage reviewers to ensure that concise and unambiguous problem descriptions are an absolute prerequisite for publication.

Because of the diversity of the problems studied in the literature, it is often difficult for any two approaches to be comparable, and extensive computational studies measuring the performance of different algorithms on the same instance sets are rare. While the diversity of applications and settings can enrich this field of study, the lack of structured comparisons makes it challenging to identify which methodological contributions deserve further investigation. Therefore, authors should always try to identify a standardized LRP variant—a variant for which benchmark instances and computational results are available—that is as close as possible to their new variant and then make a comprehensive computational comparison of their method on these instances. If this is not possible because of the lack of such a standardized and closely-related variant, we encourage authors to consider defining such standardized variants that capture the main characteristics of the practical problems they are solving in a generic fashion and to publish them together with benchmark instances and computational results, thus laying the foundation of meaningful future comparisons. New studies should also analyze which components are the most critical to the success of the proposed algorithms (e.g., via ablation studies) and, therefore, promising for tackling other problems.

Route length formulas play a vital role in approximation algorithms for LRPs and have also been considered as a potential speedup technique in heuristics (see, e.g. Albareda-Sambola, Fernández, et al. 2012). The idea is to replace the often time-consuming routing phase used in most heuristics to determine the routing costs of a given facility configuration with a fast estimate of these costs. Unfortunately, route length formulas have so far not been successful, probably due to a lack of precision in the route cost estimates. However, given the great advances in machine learning (ML) methods in recent years, transferring the underlying idea of using an estimate produced by an ML algorithm instead of employing a dedicated routing algorithm is a promising avenue for future research. A very recent paper by Sobhanan et al. (2025) follows this approach to address the standard LRP: the authors use a trained graph neural network to predict routing costs and embed it within a genetic algorithm searching for facility configurations and customer allocations. The algorithm achieves decent results on the TUZUN and the BARRETO set. The potential speedup produced by ML-based methods could also play a role in providing practitioners with improved decision support. Planning tools with graphical user interfaces that offer the possibility of quickly adjusting scenarios, problem parameters, or algorithmic parameters, heavily rely on very fast algorithms to make the interaction of the planner with the tool a pleasant experience.

LRP practitioners could also strongly benefit from simpler algorithms that provide good performance but cannot compete with the sometimes very complex state-of-the-art approaches (see, e.g., Löffler et al. 2023). Although this statement is widely acknowledged in the routing community, our personal experience tells us that it is very hard to publish such papers in good journals because of their lack of competitiveness with the best-performing algorithms. We encourage reviewers to take a more practice-oriented stance when handling such papers.

Several LRP variants included in this survey align with the United Nations Sustainable Development Goals (UNSDGs). LoRPs (Arslan 2021; Haghi et al. 2023) can be used to improve access to healthcare services (e.g., testing centers during pandemics, vaccine distribution, mobile clinics) or to guarantee enhanced accessibility to education infrastructure in underserved regions. This represents a contribution towards the UNSDG 3 "Good health and well-being" and the UNSDG 10 "Reduced inequalities". LRPs with simultaneous pickup and delivery (see, e.g., Karaoglan, Altiparmak, et al. 2011) combine forward-deliveries with reverse-logistic flows, thus contributing to the UNSDG 12 "Responsible consumption and production". LRPs minimizing emissions (see, e.g., Koç, Bektaş, et al. 2016b) directly address the UNSDG 13 "Climate Action". Cooperative LRPs are based on inter-firm partnerships that can foster more cost-efficient distribution networks and achieve meaningful CO_2 reductions (Quintero-Araujo et al. 2017), resulting in a contribution towards UNSDG 17 "Partnerships for the goals", in addition to UNSDG 12 and 13. The secure pickup and delivery LRPTW (Allahyari et al. 2021) advances the UNSDG 11 "Sustainable cities and communities" and the UNSDG 8 "Decent work and economic growth" by improving safety within the city and for drivers. The L&LRP (Arslan et al. 2021) contributes to UNSDG 3 by determining where to locate refugee camps and how to route medical- and education-service teams, and to UNSDG 11 by improving the efficiency of humanitarian assistance and disaster management.

To further advance the alignment of LRP research with the UNSDGs, we note that several promising directions have so far remained underexplored. For example, to address UNSDG 15 "Life on land", LRP variants that consider the impact on biodiversity could be studied to allow infrastructure and route planning in a biodiversity-friendly manner. In such problem variants, facility locations and vehicle routes are optimized to minimize the ecological impact by accounting for protected areas, wildlife corridors, and sensitive habitats. To address UNSDG 7 "Affordable and clean energy", an LRP variant with solar-powered facilities could be studied, in which vehicles are electric but they cannot recharge en route due to the absence of public recharging infrastructure. In such a problem variant, each candidate facility has a site-specific photovoltaic yield that reflects local shading from, e.g., close buildings, trees, or mountains. Facilities that are more exposed to the sun can collect more energy and dispatch electric vehicles with a battery autonomy that allows them to travel longer distances. These facilities are typically located on a city's outskirts, thus requiring lower opening costs, but longer travel times to reach customers located in urban centers. Facilities in partial shade (usually located in more populated areas) have higher opening costs and can only dispatch electric vehicles capable of traveling shorter distances. However, they guarantee shorter travel times to reach customers and are attractive if the areas to serve from that facility require short and energy-light routes.

Finally, the wide applicability of LRPs to real-life applications, which was already noted in previous surveys, still continues, as shown in Table 2. The table summarizes the LRP papers explicitly originating from an application or involving a case study from different industries such as retail, waste management, energy, banking, and humanitarian logistics.

Further observations and recommendations restricted to specific LRP variants are discussed in the following subsections.

Observations and recommendations for the standard location-routing problems

Although some research still focuses on the standard LRP, the trend is to study more complex and often newly defined LRP variants. This is motivated by the fact that these variants better capture real-world settings. Another reason could be that the standard LRP is very competitive: methods are comparable because of standardized benchmark instances, and many high-quality exact and heuristic solution methods have been developed over the years. Therefore, it is not surprising that only one new exact method (Liguori et al. 2023) has been recently proposed, which, however, is able to solve notably larger instances than was possible before.

On the heuristic side, we can witness a trend towards more flexible algorithms, which are able to simultaneously solve multiple LRP variants and/or related VRP variants (see, e.g., Accorsi and Vigo 2020; Arnold and Sörensen 2021; Schneider and Löffler 2019; Voigt et al. 2022). However, it has to be noted that the tackled variants are often closely related and the flexibility can be achieved by simple modifications of the input data. As already pointed out by Schneider and Drexl (2017), the medium-sized Barreto, Tuzun, and Prins benchmark sets are no longer sufficient to assess the quality of different LRP methods, and computational results on the newer Schneider benchmark set with larger and structurally different instances have been reported in several recent papers (this is also true for the exact method of Liguori et al., 2023 and the approximation algorithm of Heine et al., 2023). While it is still true that a strong and fast routing engine is a prerequisite for a competitive standard LRP method, the battleground seems to have shifted more towards finding high-quality facility configurations: Because of the very good performance of the routing algorithms used in recent methods, there is likely only a very small difference in the routing costs calculated for a specific facility configuration. Therefore, to achieve superior solution quality on more challenging instances, e.g., of the Schneider set, algorithms have to be able to identify (near-)optimal facility configurations. Obtaining notable improvements because of a superior routing, as was possible several years ago on the less demanding benchmark sets with some competitors concentrating on computing fast (but not top-quality) routing solutions, has become very unlikely. Here, the currently best-performing methods of Arnold and Sörensen (2021) and Schneider and Löffler (2019) share a similar strategy: in the beginning, facility configurations are evaluated based on quickly computed routing solutions to eliminate unpromising configurations. Then, the evaluation gets progressively more thorough, thus guaranteeing that runtime is dedicated to evaluating promising configurations. In addition, they use heuristic techniques to identify promising facility configurations based on the spatial distribution of customers and potential facility locations, costs, and capacities. We believe that combining or extending the ideas presented in these papers or developing new mechanisms for searching the space of facility configurations is a meaningful field of research that has the potential to further improve the quality of heuristic solution methods—not only for the standard LRP. To reduce the setup costs for this kind of research, a freely available high-quality VRP solver, like, e.g., the hybrid genetic search of Vidal (2022) could be employed.

Paper	**Problem type**	**Application**
Rahim and Sepil (2014)	LRP variant	Urban glass recycling in Turkey: location of bottle banks to which residents bring glass, and the revenue of a bank increases with its proximity to residents (distance-based willingness to recycle). Routing of an uncapacitated vehicle to serve opened bottle banks plus non-residential customers. The objective maximizes the revenue minus routing costs.
Tokgöz, Alwazzi, et al. (2014)	LRP on Riemannian manifold	LRP in mountainous regions such as Utah: location of gas storage facilities and vehicle routing for visiting gas wells, considering geodesic distances for route curvature.
Koç, Bektaş, et al. (2016b)	LRP with environmental considerations	Urban LRP for grid-structured cities such as Ottawa: city modeled as a grid divided by the government into speed zones (e.g., 15, 20, 25 mph). The objective minimizes facility opening, fleet composition, and emission costs. Emissions are dependent on the zone-dependent arc traveling speed, vehicle load, and engine type. The shortest arc is not necessarily the least polluting one.
Çetinkaya et al. (2018)	LRP with time windows on arcs	Terror-risk areas (e.g., Eastern Turkey) or peak-hour access restrictions in city logistics: location of facilities and routing considering time-restricted road access.
Schmidt et al. (2019)	LRP with time-dependent travel times	Urban LRP with daily traffic congestion: location of a facility and vehicle routing to minimize the total driving time. The travel time of each arc varies across time intervals representing daily congestion cycles. Instances are based on a furniture and appliances retailer and on the road network and traffic data from Québec City.
Bagheri Hosseini et al. (2019)	LRP with profits	Recycling electronic waste. The amount to be collected depends on the incentives paid to customers (which are decision variables). The collecting company can serve only part of the customers' demand and pays the corresponding part of the incentives.
Yılmaz et al. (2019)	LRP with profits	Military application involving drones operating from ships. Each drone can operate multiple routes. Because ships have limited capacities, drones must synchronize to avoid too many drones simultaneously on the same ship.
Dastaki et al. (2020)	LRP with multi-zone pricing	Zone-pricing for monopoly firms with geographically segmented markets: location of one facility per zone, vehicle routing, and setting of a zone-specific price that is dependent on the location of the opened facility. The applied price affects customers' demand. Instances are based on a razor distributor in Iran.
V. F. Yu et al. (2021)	LRPTDD	Services in which goods accumulate steadily, such as waste pickup: customers generate demand at a constant rate, so the quantity collected depends on the vehicle arrival time. Some customers must also be visited multiple times.
Arslan et al. (2021)	L&LRP	Cost-efficient placement of refugee camps and service routes during refugee crises performed by humanitarian institutions (e.g., the Turkish Red Crescent): refugee camps act as customers whose location must be determined. Service providers are routed from institution locations to provide each camp with multiple services (e.g., health-care, education).
Allahyari et al. (2021)	LRP with risk consideration	Transportation of valuables: treasury department location and vehicle routing for cash delivery and collection to/from branches and ATMs. To minimize risk, arrival times at each customer should be irregular, road segments to traverse should be different, and arrival times at each customer and the volume being carried over the travel distance are minimized. Case study based on a bank in Iran.
Shang, Ma, Y. Liu, and Sun (2022)	LRP with vehicle queuing time	Sorted waste collection: facility location and vehicle routing for sorted waste collection. Customers may need to be served by more than one vehicle, which causes queuing time. Case study based on a sorted-waste transportation problem in Shanghai, China.
V. Yu, Aloina, et al. (2022)	LRP with open facility number regional requirements	Regional waste disposal: location of a minimum number of disposal facilities for each region and vehicle routing. Case study based on Bandung City, Indonesia.
Shang, Ma, and Y. Liu (2023)	LRP with multi-compartment vehicles	Source-separated waste collection with vehicle size compartment decisions. Case study based on the source-separated waste transportation problem in Shanghai, China.
Rave and Fontaine (2025)	LRP with cargo bike routes	Use of cargo-bikes in last-mile delivery in hilly regions: location of micro hubs and routing of cargo bikes. The travel speed of bikes depends on their remaining load and the road slope. Locating hubs in valleys yields longer travel times. Instances based on five cities with different average road gradients.
Kemmar et al. (2025)	LRP variant	Servicing refugee camps in Lebanon. Some camps (customers) must be served from two hubs to create redundancy in case facilities or roads (arcs) are disrupted and alternative paths must be sought.

Table 2: Applications and case studies of the LRP variants considered in this survey.

Observations and recommendations for location-routing problems with pickup and delivery

In the LRPPD literature, the predominant application areas are freight and parcel logistics. Very few LRPPD papers, such as Fallah-Tafti et al. (2022), are focused on transporting people. On the contrary, in the literature on VRPs involving pickups and deliveries, problems involving transporting people (called dial-a-ride problems) are widely studied. Regarding the HLPR, a few papers mention planning hub-and-spoke transit networks as a possible application, but no paper explicitly focuses on this topic. In our opinion, the fact that the HLRP explicitly models flows between customers makes it suitable for the strategic modeling of public transport systems and could be a promising research avenue.

Several LRPPDs focus on the last mile of the supply chain and use a high level of detail, explicitly modeling synchronization, time windows, or open routes. Yet, we note the absence of other relevant real-world aspects like loading constraints, sorting parcels at facilities, or park-and-loop delivery, which are increasingly common in the VRP literature. It is unclear if disregarding some of these operational considerations is an acceptable simplification for a strategic-level problem, or, on the contrary, if it can cause long-term adverse effects by creating routes diverging significantly from the ones implemented in practice.

Finally, we were surprised by the lack of works focusing on supply chains in which customer orders must be fulfilled from multiple facilities. Large online marketplaces (e.g., Amazon and Alibaba) sell such a large variety of goods that it is possible that a customer order cannot be fulfilled from its assigned facility because the purchased object is stored at a different facility. In multi-product orders, different products can be stored at different facilities and are usually consolidated before being shipped. The resulting problem is not a two-echelon LRP because it lacks a central depot, nor is it an HLRP because the origins of the requests are not customers. A possible reason why this problem has not been considered in the LRP literature is that multi-product orders requiring fulfillment from multiple facilities only represent a small fraction of the total order volume. However, it is unclear if this fraction can disproportionately increase logistic costs if ignored during strategic planning, and further research could shed light on the relevance of this application case.

Observations and recommendations for other location-routing problems

Concerning LLRPs, we note that a lot of papers make meaningful comparisons to previously published works on established benchmark instances. Currently, the authors restrict themselves to the medium-sized PRINS, TUZUN, and BARRETO sets, however, tackling the SCHNEIDER set interpreted as LLRP instances could spark methodological innovations. Regarding LRPs with environmental consideration, we note that most works consider an urban setting and minimize or limit emissions in cities. However, with the continuous trend of electrification of urban vehicles, environmental considerations become more meaningful for long-distance transportation.

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