Online appendix
Recent Developments in Location-Routing Problems

Deterministic, single-echelon, single-objective, single-period problems

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September 23, 2025

A Mathematical models of the standard location-routing problem and selected variants

In this section, we present mathematical models for the standard location-routing problem (LRP) and selected LRP variants that are well-studied in the literature. Table 1 summarizes the notation used in the following mathematical formulations.

A.1 Standard location-routing problem

We present a two-index formulation for the standard LRP (Section 2 in the survey) based on the model of Löffler et al. (2023). We denote the set of facility locations by I, the set of customer locations by J, and the set of all locations by $V = I \cup J$. The (asymmetric) cost for traveling from node $i \in V$ to $j \in V \setminus \{i\}$ is represented by c_{ij}. Each facility location $i \in I$ has an opening cost O_i and a capacity of W_i units. An unlimited fleet of identical vehicles is available at each facility location, and the fixed costs and the capacity of the vehicles are denoted as F and Q, respectively. Each customer $j \in J$ has a demand of d_j units. The binary variable x_{ij} is equal to one if a vehicle travels from node $i \in V$ to $j \in V \setminus \{i\}$ and zero otherwise. Moreover, we introduce the binary variable w_{ij}, which is equal to one if customer $j \in J$ is served by facility $i \in I$ and zero otherwise. Finally, the binary variable y_i indicates whether facility $i \in I$ is opened or not.

Using the above notation, the standard LRP can be formulated as follows:

$$\min \quad \sum_{i \in V} \sum_{j \in V \setminus \{i\}} x_{ij} c_{ij} + \sum_{i \in I} \sum_{j \in J} x_{ij} F + \sum_{i \in I} y_i O_i \tag{1a}$$

$$\text{subject to} \quad \sum_{i \in V \setminus \{j\}} x_{ij} = 1 \qquad \forall j \in J \tag{1b}$$

$$\sum_{i \in V \setminus \{j\}} x_{ji} = 1 \qquad \forall j \in J \tag{1c}$$

$$\sum_{i \in I} w_{ij} = 1 \qquad \forall j \in J \tag{1d}$$

Sets	**Definition**
I	Set of facility locations.
J	Set of customer locations.
$V = I \cup J$	Set of all locations.
K	Set of vehicles.
Parameters	**Definition**
c_{ij}	Cost for traveling from location $i \in V$ to $j \in V \setminus \{i\}$.
$\bar{c}_{ij}^{lm}$	Cost for transporting the demands from i to j and from j to i ($i, j \in V$, $i \neq j$) on facility-to-facility edge (l, m) ($l, m \in V$, $l \neq m$).
δ_{ij}	Travel time from location $i \in V$ to $j \in V \setminus \{i\}$ (including service time at location i).
O_i	Opening cost of facility $i \in I$.
W_i	Capacity of facility $i \in I$.
F	Vehicle fixed cost.
Q	Vehicle capacity.
Δ	Maximum route duration.
π_j	Profit collected when serving customer $j \in J$.
d_j	Demand to be delivered at customer $j \in J$.
p_j	Demand to be picked up at customer $j \in J$.
f_{ij}	Demand from location $i \in V$ to location $j \in V \setminus \{i\}$.
β_{ij}	Cost of assigning location $i \in V$ to location $j \in V$, when j acts as a facility serving customer i.
$[a_i, b_i]$	Time window of location $i \in V$.
M	A big number.
Variables	**Definition**
$x_{ij} \in \{0, 1\}$	One if a vehicle travels directly from location $i \in V$ to $j \in V \setminus \{i\}$, zero otherwise.
$w_{ij} \in \{0, 1\}$	One if customer $j \in J$ is served by facility $i \in I$, zero otherwise.
$s_{ij} \in \{0, 1\}$	One if customer $j \in J$ is the only customer served by facility $i \in I$, zero otherwise.
$y_i \in \{0, 1\}$	One if facility $i \in I$ is opened, zero otherwise.
$t_l \geq 0$	Arrival time/Start of visiting time at location $l \in V$.
$u_{ij} \geq 0$	Delivery demand for customers following $i \in J$ carried on the vehicle when it travels directly from i to $j \in V$; zero if the vehicle does not travel directly from i to j.
$v_{ij} \geq 0$	Pickup demand for customers up to $i \in J$ carried on the vehicle when it travels directly from i to $j \in V$; zero if the vehicle does not travel directly from i to j.
$\varphi_{ij}^{lm} \geq 0$	Fraction of demand $f_{ij} + f_{ji}$ ($i, j \in V$, $i < j$) routed on facility-to-facility edge (l, m) ($l, m \in V$). If i and j are served from the same facility l, $\varphi_{ij}^{ll} = 1$.
$q_{lk} \in \{0, 1\}$	One if vehicle $k \in K$ is assigned to location $l \in V$, zero otherwise.
$\eta_{ij}^{lk} \in \{0, 1\}$	One if vehicle $k \in K$ is assigned to location $l \in V$ and travels directly from location $i \in V$ to $j \in V \setminus \{i\}$, zero otherwise.
$\xi_i^{lk} \in \{0, 1\}$	One if location $i \in V$ is serviced by vehicle $k \in K$ assigned to location $i \in V$.
$\psi_{ij}^{lkm} \geq 0$	In the multi-commodity flow formulation of the HLRP, it denotes the fictitious flow sent along edge (i, j) ($i, j \in V$, $i \neq j$) if vehicle $k \in K$ is assigned to location $l \in V$ and serves location $m \in V$.

Table 1: Summary of the mathematical notation.

$$
\begin{aligned}
& w_{ij} \leq y_i && \forall i \in I,\ \forall j \in J && \text{(1e)}\\
& \sum_{j \in J} w_{ij} d_j \leq W_i && \forall i \in I && \text{(1f)}\\
& x_{jk} \leq 1 - w_{ij} + w_{ki} && \forall i \in I,\ \forall j \in J,\ \forall k \in J && \text{(1g)}\\
& x_{jk} \leq 1 + w_{ij} - w_{ki} && \forall i \in I,\ \forall j \in J,\ \forall k \in J && \text{(1h)}\\
& x_{ij} \leq w_{ij} && \forall i \in I,\ \forall j \in J && \text{(1i)}\\
& x_{ji} \leq w_{ij} && \forall i \in I,\ \forall j \in J && \text{(1j)}\\
& \sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ij} \leq |S| - \left\lceil \frac{1}{Q} \sum_{i \in S} d_i \right\rceil && \forall S \subseteq J && \text{(1k)}\\
& x_{ij} \in \{0,1\} && \forall i \in V,\ \forall j \in V \setminus \{i\} && \text{(1l)}\\
& w_{ij} \in \{0,1\} && \forall i \in I,\ \forall j \in J && \text{(1m)}\\
& y_i \in \{0,1\} && \forall i \in I. && \text{(1n)}
\end{aligned}
$$

The objective function (1a) minimizes the sum of total routing cost, vehicle fixed cost, and facility opening cost. Constraints (1b) and (1c) guarantee that every customer is served exactly once, and constraints (1d) force every customer to be assigned to exactly one facility. Constraints (1e) ensure that customers can only be served by open facilities, and constraints (1f) guarantee that the facility capacity is respected. Constraints (1g) and (1h) allow traveling along an arc between two customers only if both customers are assigned to the same facility. According to constraints (1i) and (1j), a route starts and ends at the facility that serves the customers on the respective route. Constraints (1k) are extended subtour elimination constraints, which also serve as vehicle capacity constraints. Finally, the binary decision variables are defined in constraints (1l)–(1n).

A.2 Location-routing problem with simultaneous pickup and delivery

We present a two-index formulation for the LRP with simultaneous pickup and delivery (Section 3.1.1 in the survey) based on the model of Karaoglan et al. (2011). The formulation presented below extends the one for the standard LRP of Section A.1 using the notation presented in Table 1.

$$
\begin{aligned}
\min\quad & \text{(1a)}\\
\text{subject to}\quad & \text{(1b)–(1e), (1i), (1j)}\\
& \sum_{j \in V} u_{ji} - \sum_{j \in V} u_{ij} = d_i && \forall i \in J && \text{(2a)}\\
& \sum_{j \in V} v_{ij} - \sum_{j \in V} v_{ji} = p_i && \forall i \in J && \text{(2b)}\\
& u_{ij} + v_{ij} \leq Q x_{ij} && \forall i \in V,\ \forall j \in V && \text{(2c)}\\
& \sum_{j \in J} u_{ij} = \sum_{j \in J} d_j w_{ij} && \forall i \in I && \text{(2d)}\\
& \sum_{j \in J} u_{ji} = 0 && \forall i \in I && \text{(2e)}\\
& \sum_{j \in J} v_{ji} = \sum_{j \in J} p_j w_{ij} && \forall i \in I && \text{(2f)}\\
& \sum_{j \in J} v_{ij} = 0 && \forall i \in I && \text{(2g)}\\
& u_{ij} \leq (Q - d_i) x_{ij} && \forall i \in J,\ \forall j \in V && \text{(2h)}
\end{aligned}
$$

$$v_{ij} \leq (Q - p_j)x_{ij} \qquad \forall i \in V,\ \forall j \in J \tag{2i}$$

$$u_{ij} \geq d_j x_{ij} \qquad \forall i \in V,\ \forall j \in J \tag{2j}$$

$$v_{ij} \geq p_i x_{ij} \qquad \forall i \in J,\ \forall j \in V \tag{2k}$$

$$\sum_{j \in J} d_j w_{ij} \leq W_i y_i \qquad \forall i \in I \tag{2l}$$

$$\sum_{j \in J} p_j w_{ij} \leq W_i y_i \qquad \forall i \in I \tag{2m}$$

$$x_{ij} + w_{ik} + \sum_{l \in V \setminus \{k\}} w_{jl} \leq 2 \qquad \forall i \in J,\ \forall j \in J \setminus \{i\},\ \forall k \in I \tag{2n}$$

(1l)–(1n)

$$u_{ij} \geq 0 \qquad \forall i \in V,\ \forall j \in V \tag{2o}$$

$$v_{ij} \geq 0 \qquad \forall i \in V,\ \forall j \in V. \tag{2p}$$

Constraints (2a) and (2b) are flow conservation constraints for the delivery and pickup flows, respectively. Constraints (2c) ensure that the vehicle capacity is respected. Constraints (2d) and (2f) ensure that the delivery (pickup) goods dispatched from a facility equal the delivery (pickup) demand of the assigned customers. Constraints (2e) force the vehicles to return to the facilities with no delivery load, and, analogously, (2g) force the vehicles to leave the facilities with no pickup load. Constraints (2h)–(2k) set upper and lower bounds on the delivery and pickup load along each arc. Constraints (2l) and (2m) ensure that the total delivery and pickup loads handled by a facility do not exceed its capacity. Constraints (2n) ensure that if two customers i and j are visited by the same vehicle, they are assigned to the same facility. Finally, constraints (2o) and (2p) define the domain of the variables.

A.3 Hub location routing problem

Although several problems fall under the umbrella of hub LRPs (Section 3.2.2 in the survey), we choose the many-to-many hub LRP (MMHLRP) because it is a popular variant general enough to capture the main tradeoffs involved in HLRPs. We present a five-index formulation for the problem based on the model of Saraiva de Camargo et al. (2013) and using the notation of Table 1. We remark that this model uses a multi-commodity flow formulation to ensure route elementarity, thus resulting in five-index flow variables ψ_{ij}^{lkm}. In the MMHLRP, any customer can be chosen to act as a facility, therefore $J = I = V$, and we use V in our model.

$$\min \quad \sum_{i \in V} O_i w_{ii} + \sum_{\substack{i,j \in V \\ i \neq j}} \beta_{ij} w_{ij} + \sum_{\substack{i,j,l \in V \\ i \neq j}} \sum_{k \in K} c_{ij} \eta_{ij}^{lk} + F \sum_{l \in V} \sum_{k \in K} q_{lk} + \sum_{\substack{i,j,l,m \in V \\ i<j, l \neq m}} \bar{c}_{ij}^{lm} \varphi_{ij}^{lm} \tag{3a}$$

$$\text{subject to} \quad \sum_{j \in V} w_{ij} = 1 \qquad \forall i \in V \tag{3b}$$

$$w_{ij} \leq w_{jj} \qquad \forall i, j \in V,\ i \neq j \tag{3c}$$

$$\sum_{m \in V} \varphi_{ij}^{lm} = w_{il} \qquad \forall i, j, l \in V,\ i < j \tag{3d}$$

$$\sum_{l \in V} \varphi_{ij}^{lm} = w_{jl} \qquad \forall i, j, m \in V,\ i < j \tag{3e}$$

$$\sum_{j \in V \setminus \{i\}} \eta_{ij}^{lk} = \xi_i^{lk} \qquad \forall i, l \in V,\ \forall k \in K \tag{3f}$$

$$\sum_{i \in V \setminus \{j\}} \eta_{ij}^{lk} = \xi_j^{lk} \qquad \forall j, l \in V,\ \forall k \in K \tag{3g}$$

$$\eta_{ij}^{lk} \leq q_{lk} \qquad \forall i,j,l \in V,\ \forall k \in K,\ i \neq j \tag{3h}$$

$$q_{lk} \leq q_{l,k-1} \qquad \forall l \in V,\ \forall k \in K \setminus \{1\} \tag{3i}$$

$$\sum_{k \in K} \xi_i^{lk} = w_{il} \qquad \forall i,l \in V,\ i \neq l \tag{3j}$$

$$\sum_{\substack{i,j \in V \\ i \neq j}} \delta_{ij} \eta_{ij}^{lk} \leq \Delta \qquad \forall l \in V,\ \forall k \in K \tag{3k}$$

$$\psi_{ij}^{lkm} \leq \eta_{ij}^{lk} \qquad \forall i,j,l,m \in V,\ \forall k \in K,\ i \neq j,\ l \neq m,\ l \neq i,\ m \neq j \tag{3l}$$

$$\sum_{j \in V \setminus \{i\}} \psi_{ij}^{ikm} = \xi_m^{ik} \qquad \forall i,m \in V,\ \forall k \in K,\ i \neq m \tag{3m}$$

$$\sum_{i \in V \setminus \{j\}} \psi_{ij}^{lkj} = \xi_j^{lk} \qquad \forall k,l \in V,\ \forall k \in K,\ j \neq l \tag{3n}$$

$$\sum_{i \in V \setminus \{j\}} \psi_{ij}^{lkm} = \sum_{i \in V \setminus \{j\}} \psi_{ji}^{lkm} \qquad \forall j,l,m \in V,\ \forall k \in K,\ l \neq m,\ j \neq l,\ j \neq m \tag{3o}$$

$$w_{ij} \in \{0,1\} \qquad \forall i,j \in V \tag{3p}$$

$$\varphi_{ij}^{lm} \geq 0 \qquad \forall i,j,l,m \in V,\ i < j \tag{3q}$$

$$q_{lk} \in \{0,1\} \qquad \forall l \in V,\ \forall k \in K \tag{3r}$$

$$\eta_{ij}^{lk} \in \{0,1\} \qquad \forall i,j,l \in V,\ \forall k \in K,\ i \neq j \tag{3s}$$

$$\xi_i^{lk} \in \{0,1\} \qquad \forall i,l \in V,\ \forall k \in K \tag{3t}$$

$$\psi_{ij}^{lkm} \geq 0 \qquad \forall i,j,l,m \in V,\ \forall k \in K,\ i \neq j. \tag{3u}$$

The objective function (3a) minimizes the cost of opening facilities, assigning customers, using vehicles, and performing both local routes (those starting and ending at a given facility and serving customers) and the Hamiltonian tour linking the open facilities. Constraints (3b) ensure that each customer is assigned to exactly one facility. Customers chosen as facilities are assigned to themselves ($w_{ii} = 1$). Constraints (3c) state that a customer can be assigned to a facility only if the facility is open. Constraints (3d) and (3e) link the φ and w variables, stating that the i–j flow must pass through the facility to which these customers are assigned. Constraints (3f) and (3g) ensure that locations are visited by their assigned vehicles. Variables η and q are linked through constraints (3h), while constraints (3i) break symmetry in the vehicle assignment. Constraints (3j) link the ξ and w variables, ensuring that each customer is visited with a vehicle from the facility it was assigned to. The maximum route duration is enforced via constraints (3k). Constraints (3l)–(3o) define a multi-commodity flow formulation for the local routes, which ensures (with a polynomial number of constraints) that they are elementary. In particular, when $\psi_{ij}^{lkm} = 1$ (i.e., there is a flow of one unit), vehicle k assigned to hub l uses edge (i,j) to serve customer m. Finally, constraints (3p)–(3u) define the domain of the variables.

A.4 Prize-collecting capacitated location routing problem

We present a two-index formulation for the prize-collecting capacitated location routing problem (Section 4 in the survey) based on the model of Negrotto and Loiseau (2021).

$$\min \quad \sum_{\substack{i,j \in V \\ i \neq j}} c_{ij} x_{ij} + 2 \sum_{i \in I} \sum_{j \in J} c_{ij} s_{ij} + \sum_{i \in I} O_i y_i + \frac{F}{2} \sum_{i \in I} \sum_{j \in J} x_{ij} + F \sum_{i \in I} \sum_{j \in J} s_{ij} - \sum_{j \in J} \pi_j \sum_{iinI} w_{ij} \tag{4a}$$

$$\sum_{i \in V \setminus \{j\}} x_{ij} + 2 \sum_{i \in I} s_{ij} \leq 2 \qquad \forall j \in J \tag{4b}$$

$$\sum_{i\in S}\sum_{s\in S\setminus\{i\}} x_{ij} \le \sum_{i\in I}\sum_{j\in S} w_{ij} - \frac{1}{Q}\sum_{j\in S} d_j \sum_{i\in I} w_{ij} \qquad \forall S \subseteq J \tag{4c}$$

$$x_{ij} + s_{ij} \le y_i \qquad \forall i \in I,\ \forall j \in J \tag{4d}$$

$$x_{ij} + s_{ij} \le w_{ij} \qquad \forall i \in I,\ \forall j \in J \tag{4e}$$

$$\sum_{i\in I}(x_{ij} + y_{ij}) \le 1 \qquad \forall j \in J \tag{4f}$$

$$\sum_{j\in J} d_j w_{ij} \le W_i z_i \qquad \forall i \in I \tag{4g}$$

$$\sum_{i\in I\setminus\{j\}} x_{ij} + 2\sum_{i\in I} s_{ij} = 2\sum_{i\in I} w_{ij} \qquad \forall j \in J \tag{4h}$$

$$x_{jl} \le 1 - l(w_{ij} - w_{il}) \qquad \forall i \in I,\ \forall j,l \in J,\ j \ne l \tag{4i}$$

(1l), (1m), (1n)

$$s_{ij} \in \{0,1\} \qquad \forall i \in I,\ \forall j \in J. \tag{4j}$$

The objective function (4a) sums the routing costs, the facility opening costs, and the fixed vehicle costs (while taking into account the special back-and-forth routes defined by variables s_{ij}), minus the profit collected at the visited customers. Constraints (4b) ensure that each customer is visited at most once. Constraints (4c) enforce that the capacities of the vehicles are respected. Constraints (4d) ensure that customers are only visited from open facilities. Constraints (4e) make sure that a customer is visited either as part of a multi-stop route (defined by variables x) or with a back-and-forth route (defined by variables s) but not both. Constraints (4f) impose that routes start and end at the same facility. Facility capacity constraints are enforced via (4g). Constraints (4h) ensure that an assigned customer is visited by some vehicle. Constraints (4i) link the x and w variables. Finally, constraints (4j) define the domain of the variables.

A.5 Location-routing problem with time windows

We present a two-index formulation for a simplified variant of the LRP with time windows (Section 5 in the survey), in which we assume that the fleet of vehicles is shared among all facilities to avoid a three-index formulation. The formulation extends the one for the standard LRP of Section A.1 using the notation presented in Table 1.

$$\min \quad (1a) \tag{5a}$$

$$\text{subject to} \quad (1b) - (1n) \tag{5b}$$

$$t_i + \delta_{ij} - t_j \le M(1 - x_{ij}) \qquad \forall i \in V,\ \forall j \in V \setminus \{i\} \tag{5c}$$

$$a_l \le t_l \le b_l \qquad \forall l \in V \tag{5d}$$

$$t_l \ge 0 \qquad \forall l \in V. \tag{5e}$$

Constraints (5c) ensure that if a vehicle starts serving location i at time t_i, serves location i, and travels the arc i to j, the time that the vehicle starts serving customer j is greater or equal to the sum of t_i and travel time from location i to j (including service time at location i). Constraints (5d) are the time window constraints, guaranteeing that location l is served within the specified time window. Finally, constraints (5e) define the domain of the decision variables t_l.

A.6 Latency location-routing problem

We present a two-index formulation for a simplified variant of the latency LRP (LLRP; Section 6 in the survey), in which we assume a uniform fleet of vehicles to avoid a three-index formulation.

The formulation extends the one of the standard LRP of Section A.1 using the notation presented in Table 1.

$$
\begin{aligned}
\min \quad & \sum_{j \in J} t_j && && (6a) \\
\text{subject to} \quad & (1b)\text{–}(1e),\ (1g)\text{–}(1n) \\
& t_i + \delta_{ij} - t_j \leq M(1 - x_{ij}) && \forall i \in V,\ \forall j \in V \setminus \{i\} && (6b) \\
& \sum_{i \in I} y_i \leq N && && (6c) \\
& t_l \geq 0 && \forall l \in V. && (6d)
\end{aligned}
$$

The objective function (6a) minimizes the total latency, i.e., the sum of the arrival time of the vehicles at customers. Constraints (6b) ensure that if a vehicle arrives at location i at time t_i, serves location i, and travels the arc i to j, the arrival time of the vehicle at customer j is equal to the sum of t_i and the travel time from location i to j. Constraints (6c) restrict the total number of open facilities, and constraints (6d) define the domain of the decision variables t_l.

B Summary tables

In this section, we present tables that summarize the key features of the reviewed studies for the standard LRP and each major LRP variant (i.e., LRP with pickup and delivery, hub LRP, LRP with profits, LRPTW, and LLRP). In the tables, column "F?" reports on whether the paper includes a mathematical formulation for the problem.

Paper	Novelty	F?	Approach	Contribution
Liguori et al. 2023	State-of-the-art exact algorithm using a new family of nonrobust valid inequalities.	Yes	Exact (BCP)	Methodological
Granada et al. 2019	State-of-the-art formulation for the OLRP based on the graph structure of the solution (spanning forest consisting of one tree per open facility).	Yes	Exact (CPLEX)	Methodological
Arnold and Sörensen 2021	State-of-the-art heuristic based on a progressive filtering mechanism to eliminate unpromising facility configurations	No	Heuristic (progressive filtering + KGLS).	Methodological
Accorsi and Vigo 2020	Fast, versatile, and competitive heuristic for the standard LRP with uncapacitated facilities.	No	Heuristic (ILS + postoptimization based on set partitioning)	Methodological
Löffler et al. 2023	Conceptually simple heuristic leading to reasonable results, impact assessment of algorithmic components, effective facility configuration refinement phase.	Yes	Heuristic (GRASP + VNS)	Methodological
Voigt et al. 2022	Competitive hybrid heuristic using multiple heuristic components.	No	Heuristic (GA + ALNS)	Methodological
Ferreira and Queiroz 2018	Heuristic with diversification based on randomly opening and closing facilities and using SA, competitive results obtained for TUZUN instances.	Yes	Heuristic (SA)	Methodological
Akpunar and Akpinar 2021	Hybrid heuristic using ALNS for diversification and VNS for intensification and obtaining competitive results on BARRETO instances.	No	Heuristic (ALNS + VNS)	Methodological
Yu et al. 2019	GA with a new crossover operator obtaining competitive results on DUHAMEL instances.	No	Heuristic (GA)	Methodological
Gørtz and Nagarajan 2016	First constant-factor approximation algorithm for the k-LRP, introduces the new k-median forest problem.	No	Approximation	Methodological
Heine et al. 2023	First constant-factor approximation algorithm for the standard LRP with arbitrary facility capacities.	Yes	Approximation	Methodological

Table 2: Summary of reviewed papers for the standard LRP.

Paper	Novelty	F?	Approach	Contribution
Capelle et al. (2019)	Route-based extended formulation and B&P.	Yes	Exact (B&P)	Methodological
Domínguez-Martín et al. (2024)	B&C for one-commodity pickup–delivery LRP.	Yes	Exact (B&C)	Methodological
Gianessi et al. (2016)	Introduces Ring LRP with open routes and a facility ring; exponential-sized model.	Yes	Matheuristic (set-partitioning + heuristics)	Methodological
Lopes et al. (2014)	Defines MMpHLP and releases benchmark set.	Yes	Heuristics	Methodological (benchmark)
De Freitas et al. (2023)	BRKGA for the MMpHLP with directed tours.	No	Heuristic (BRKGA)	Methodological
Pandiri and Singh (2021)	Hyper-heuristic for the MMpHLP combining three simple heuristics.	No	Heuristic (hyper-heuristic)	Methodological
Ratli et al. (2020)	Variable neighborhood search heuristic for the HLRP.	No	Heuristic	Methodological
Ghaffarinasab et al. (2018)	Continuous-location HLRP for strategic design with approximate routing costs.	Yes	Approximation + heuristics	Application-oriented (postal services)
Wu et al. (2022)	ALNS for multi-allocation HLRP with split pickup and delivery.	No	Heuristic (ALNS)	Methodological
Yıldız et al. (2021)	Exact (B&C) for an HLRP in which facilities are linked through arbitrary paths.	Yes	B&C	Methodological

Table 3: Summary of reviewed papers for the LRP with pickup and delivery and the hub LRP.

Paper	Novelty	F?	Approach	Contribution
Negrotto and Loiseau (2021)	B&C with strong valid inequalities for prize-collecting capacitated LRP.	Yes	Exact (B&C)	Methodological
Ahmadi-Javid et al. (2018)	Extended formulation and B&P algorithm for a profit-maximization LRP with price-sensitive customers	Yes	Exact (B&P)	Methodological.
Bagheri Hosseini et al. (2019)	Incentive-driven e-waste LRP with partial collection; ILS with problem-specific moves.	Yes	Heuristic (ILS)	Application-oriented
Yakıcı (2016)	OP-style location–routing for military targets; formulations and heuristics.	Yes	Heuristic	Application-oriented
Nadizadeh (2021)	Formulation and heuristic for the orienteering LRP.	Yes	Heuristic	Methodological
Yılmaz et al. (2019)	UAV location–routing with spatio-temporal synchronization.	No	Heuristic (ACO)	Application-oriented

Table 4: Summary of reviewed papers for the LRP with profits.

Paper	**Novelty**	**F?**	**Approach**	**Contribution**
Ponboon et al. 2016	First exact approach for the LRPTW.	Yes	Exact (B&P)	Methodological
Farham et al. 2018	Heuristically-derived upper bounds accelerate the exact algorithm.	Yes	Exact (B&P); Heuristic (hierarchical)	Methodological
Koç et al. 2016	LRPTW extension (fleet size and mix), integration of fleet sizing and location decisions in the removal and insertion operators.	Yes	Heuristic (GA+ALNS)	Application-oriented
Çetinkaya et al. 2018	LRPTW variant (time windows on arcs).	Yes	Exact (CPLEX)	Application-oriented
Rave and Fontaine 2025	LRPTW variant (load-dependent travel times).	Yes	Heuristic (ALNS)	Application-oriented

Table 5: Summary of reviewed papers for the LRPTW.

Paper	**Novelty**	**F?**	**Approach**	**Contribution**
Moshref-Javadi and Lee 2016	First study on LLRP, evaluation of the impact of considering latency, bounding procedures, first heuristic approach for the LLRP.	Yes	Heuristic (MA)	Application-oriented; Methodological
Nucamendi-Guillén et al. 2022	First exact approaches for the LLRP, state-of-the-art mathematical formulations, LLRP extension (facility opening costs).	Yes	Exact (B&C, branch-and-check); Heuristic (GRASP+ILS)	Application-oriented; Methodological
Osorio-Mora, Rey, et al. 2023	Analysis of three different VND-based intensification strategies for a heuristic delivering improved solutions.	No	Heuristic (SA+VND)	Methodological
Osorio-Mora, Escobar, et al. 2023	Competitive heuristic based on obtaining already good-quality starting solutions with a sophisticated construction heuristic.	No	Heuristic (ILS+SA+VND)	Methodological
Zou et al. 2024	State-of-the-art heuristic for the LLRP using multi-parent edge assembly crossover and reinforcement learning to determine the neighborhood exploration order.	No	Heuristic (MA + reinforcement learning)	Methodological

Table 6: Summary of reviewed papers for the LLRPs.

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