Deriving intersection cuts from wide split disjunctions

Sven Wiese [1]

with Pierre Bonami [2], Andrea Lodi [1]
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[1]DEI, University of Bologna

[2]CPLEX Optimization

Bologna, 30/04/2015

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@OR_Unibo

A motivating example...

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A motivating example...

$x_{1,1}$

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$D(x_{1,1}) = \{1,2,3,4,5,6,7,8,9\}$

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$$1 \leq x_{1,1} \leq 9$$

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$$4 \leq x_{1,1} \leq 6$$

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8	4	5	3	6	2	7	9	9
1	3	7	5	9	4	8	2	6
2	7	8	6	4	3	5	1	9
3	9	1	2	8	5	6	7	4
4	5	6	7	1	9	2	8	3
9	6	3	8	5	7	1	4	2
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Instead, we generate cutting planes from the disjunctions associated with the holes in the integer variable domains. We call these disjunctions wide split disjunctions.

Such an approach can as well be extended to continuous variables. In this talk, we restrict to integer variables.

Outline

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Variables with non-contiguous domains

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In this talk we consider a Mixed Integer Linear Program (MILP) in the general form

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax = b \\ & x \in \mathbb{Z}_+^p \times \mathbb{R}_+^n, \end{aligned} \tag{M}$$

where $A \in \mathbb{R}^{m \times (p+n)}$, $b \in \mathbb{R}^m$.

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where $A \in \mathbb{R}^{m \times (p+n)}$, $b \in \mathbb{R}^m$.

Furthermore, we assume that there are integer variables that have non-contiguous domains, i.e., for a given variable x_i with domain $D_i = [L_i, U_i]$, there is at least one $t \in \mathbb{Z} \cap (L_i, U_i)$ such that x_i cannot be equal to t.

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In other words, the variable has a "hole" in its domain.

Variables with non-contiguous domains

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Formally, we say that $H = [h_l, h_u]$ with $(h_l, h_u) \in \mathbb{Z}^2$ is a hole for the integer variable x_i, if $H \subseteq D_i$ and if $\forall\ t \in \mathbb{Z} \cap \text{int}(H)$, we have that $x_i \neq t$ in any feasible (or optimal) solution of (M).

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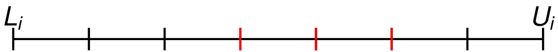

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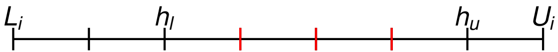

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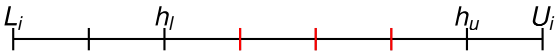

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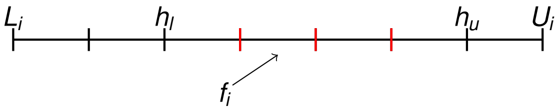

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$$x_i = \sum_{j \in K} \lambda_j x_j$$
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for some $i \in \{1, \ldots, p\}$, some subset $K \subseteq \{1, \ldots, p\}$ and $\lambda_j \in \mathbb{N}$, $j \in K$.

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for some $i \in \{1, \ldots, p\}$, some subset $K \subseteq \{1, \ldots, p\}$ and $\lambda_j \in \mathbb{N}$, $j \in K$.

If the $\lambda_j \in \mathbb{N}$ are non-consecutive, there is a hole in the domain of the integer variable x_i.

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The condition $x_i \in \bigcup_{j=1}^{q} [l_j, u_j]$ with $u_j + 1 < l_{j+1} \ \forall \ j = 1, \ldots, q$ can be modeled through bigM constraints:

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$$\begin{aligned} l_j - x_j &\leq (1 - x_j) \cdot (l_j - l_1) \\ x_j - u_j &\leq (1 - x_j) \cdot (u_q - u_j) \qquad \forall \; j = 1, \ldots, q \\ \sum_{j=1}^{q} x_j &= 1. \end{aligned}$$

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This constraint structure can be found, e.g., in straightforward models for the well-known Traveling Salesman Problem with Multiple Time Windows (TSPMTW).

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In such a situation, the time window of node i could be strengthened to $\{R_i\} \cup [K_i, D_i]$ for some $K_i > R_i + 1$.

Outline

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The above system defines a polyhedron $P(B)$.

To derive a cut, take a convex lattice-free set S that has the basic solution f in its interior and compute the interesction of the extreme rays of $P(B)$ with the boundary of S.

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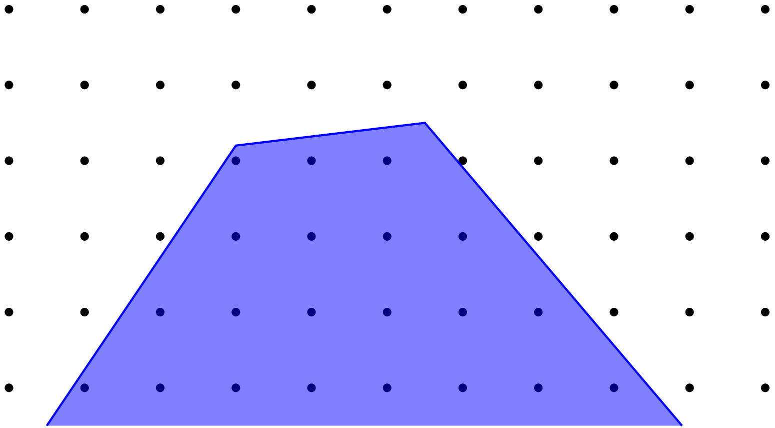

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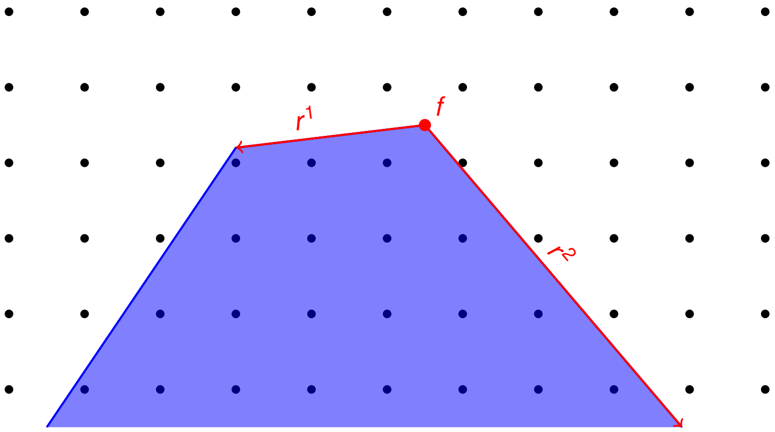

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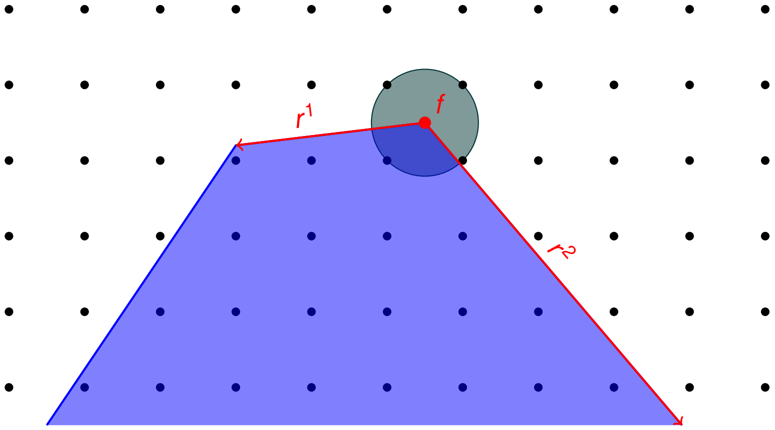

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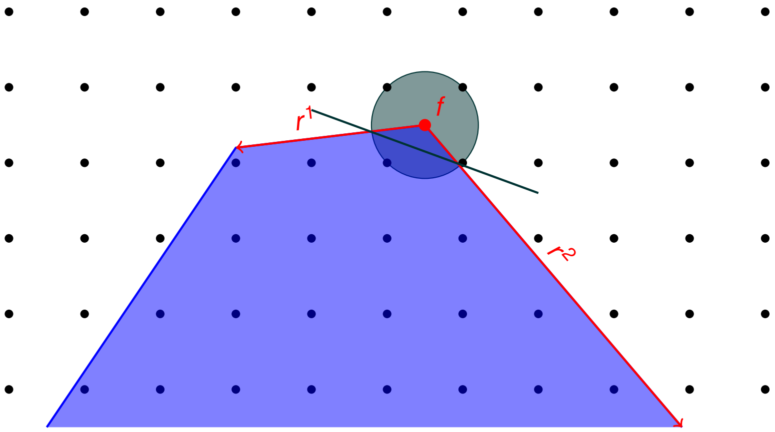

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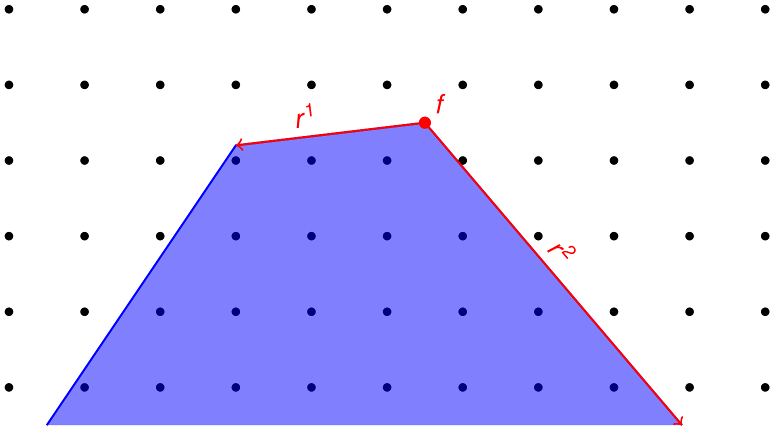

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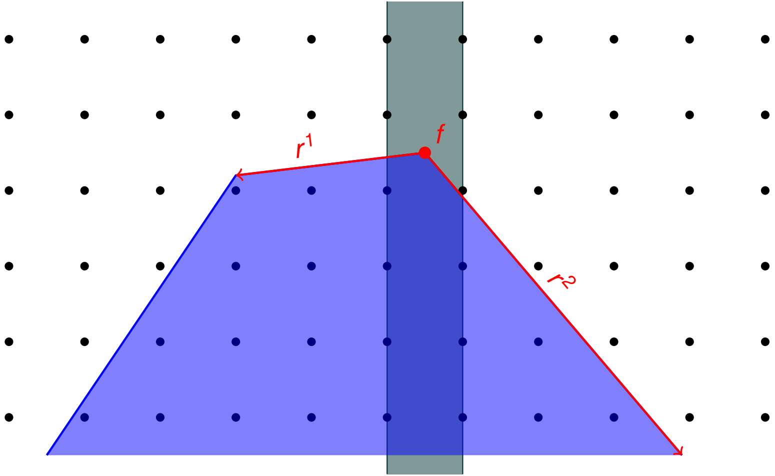

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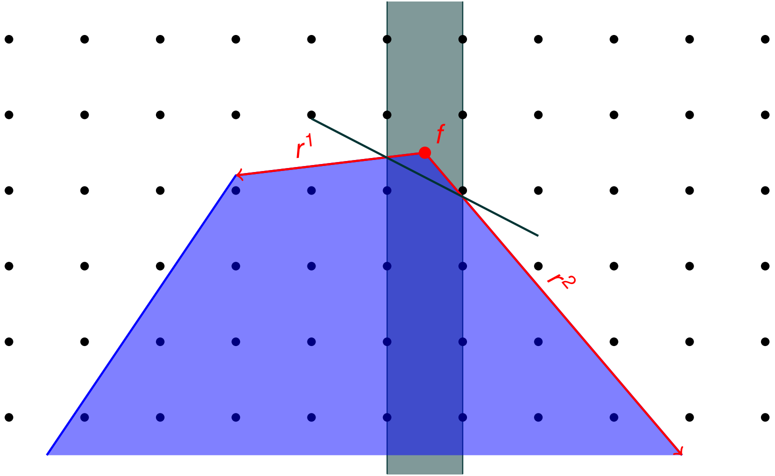

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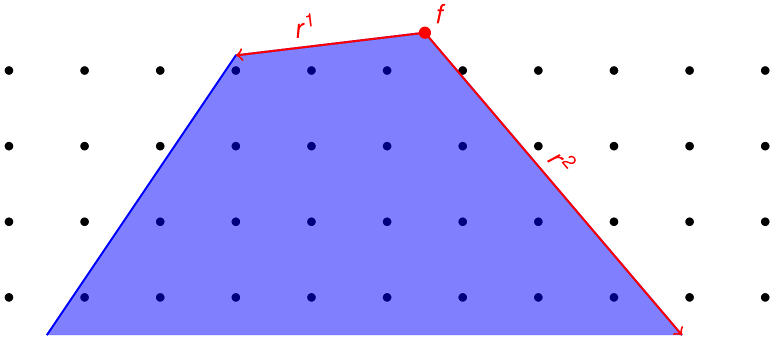

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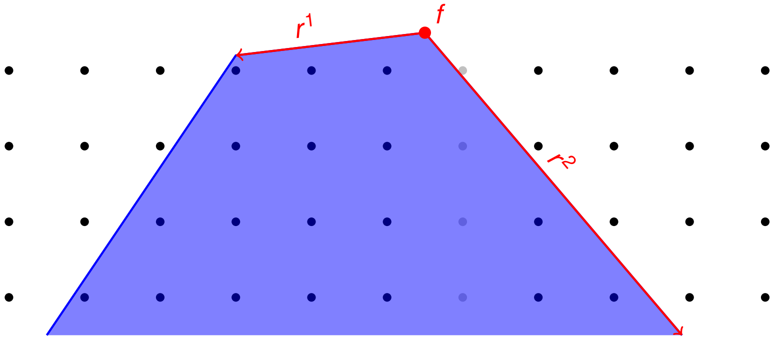

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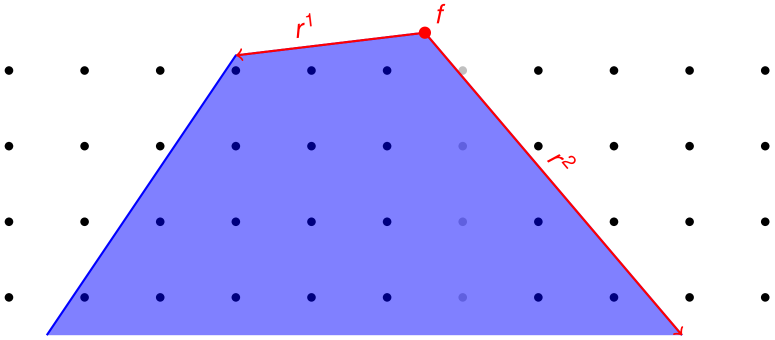

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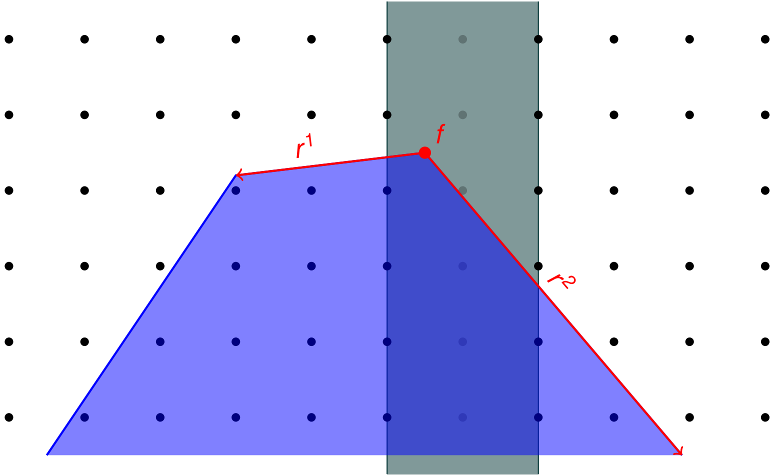

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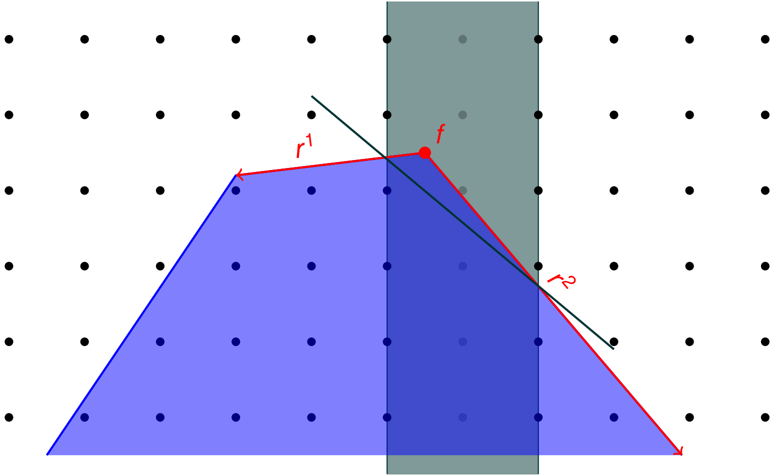

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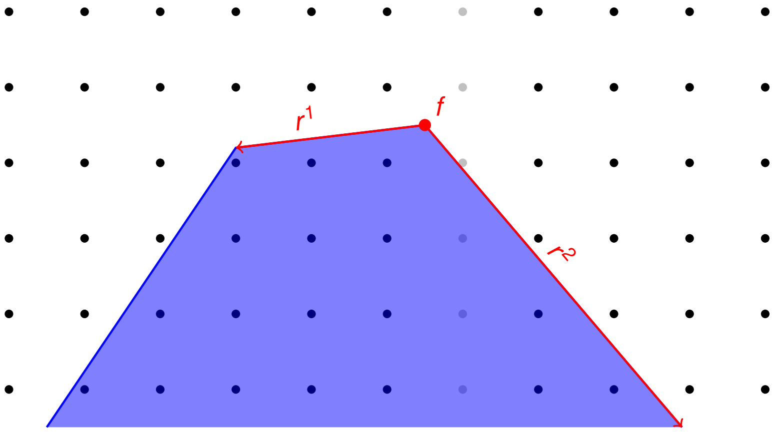

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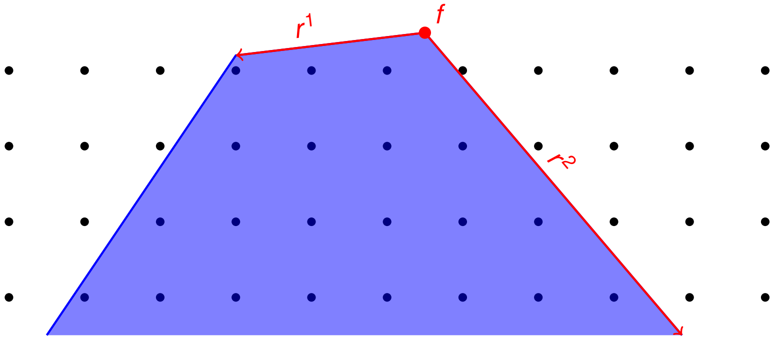

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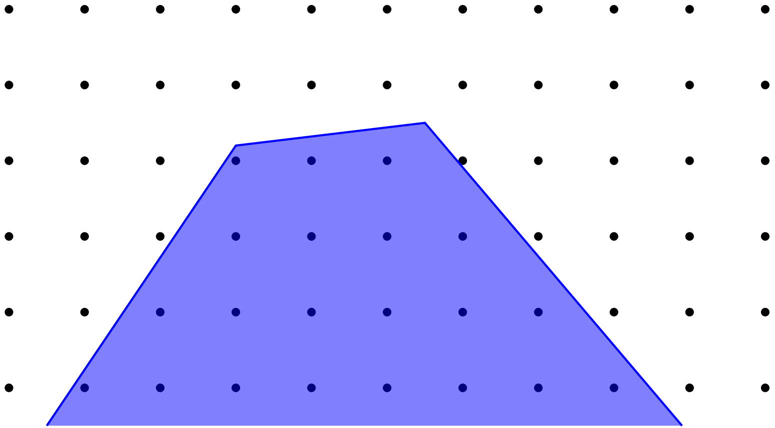

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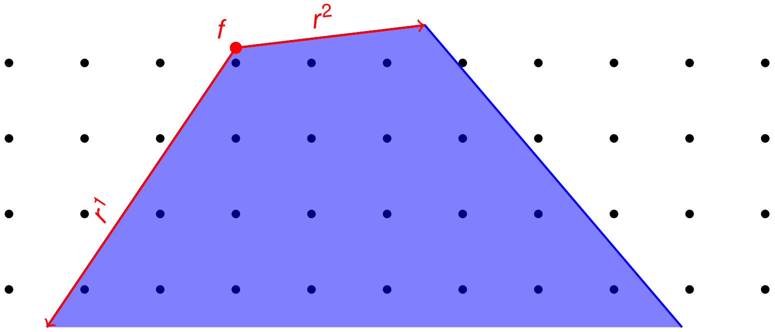

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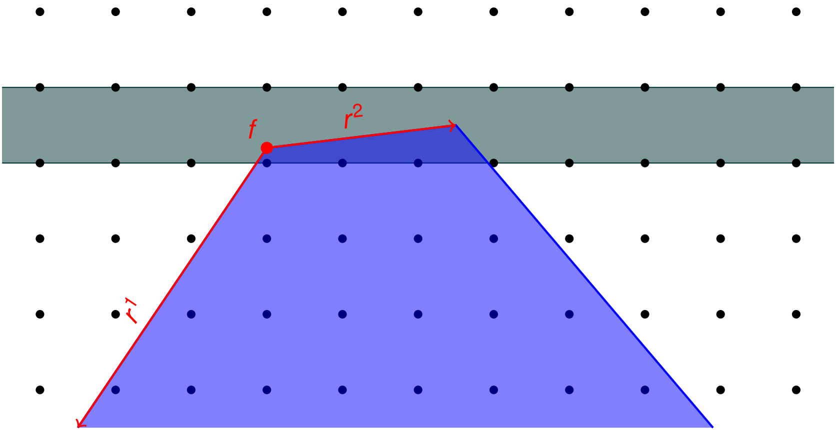

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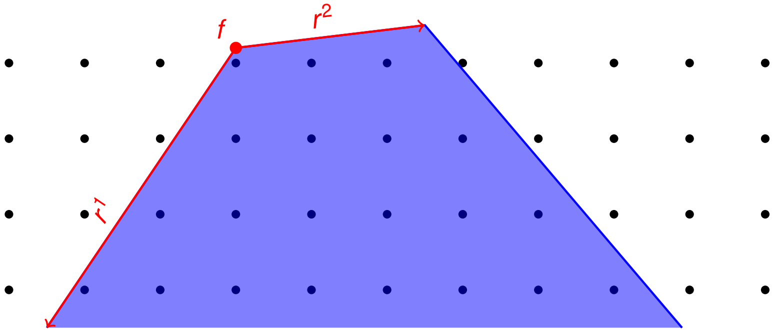

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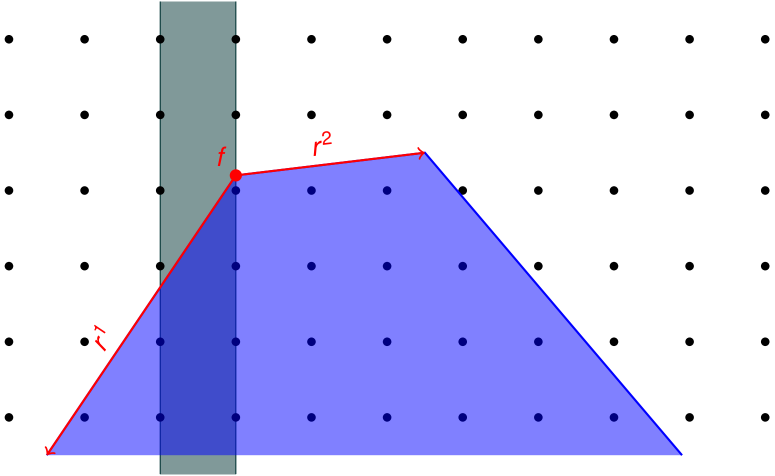

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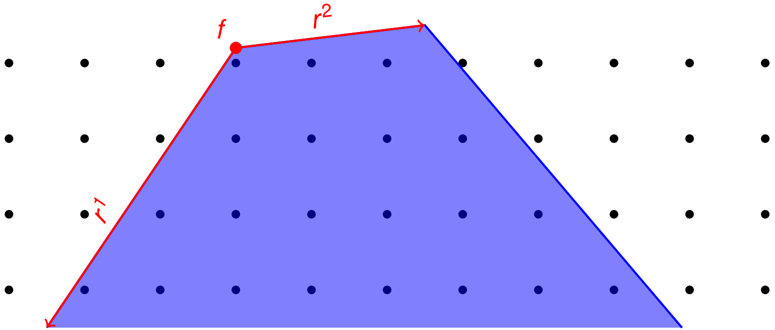

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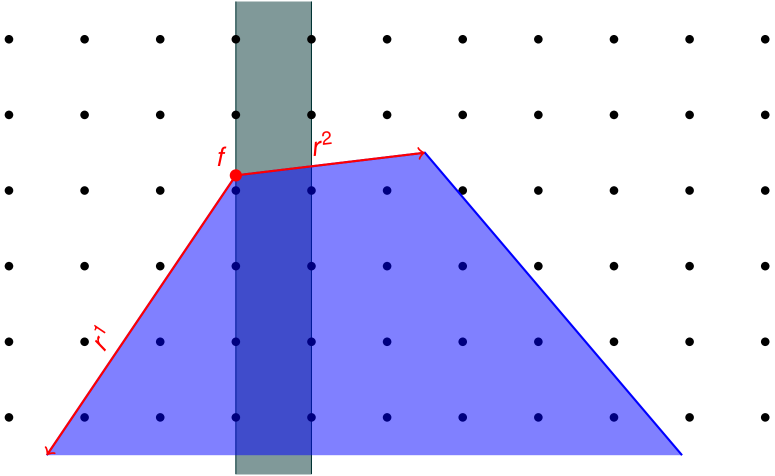

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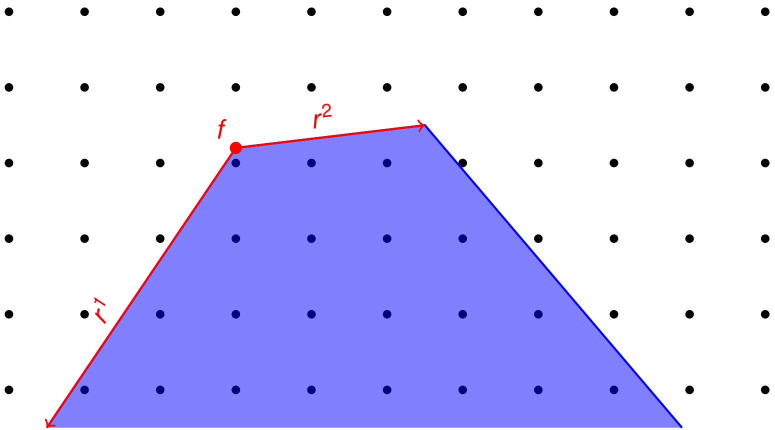

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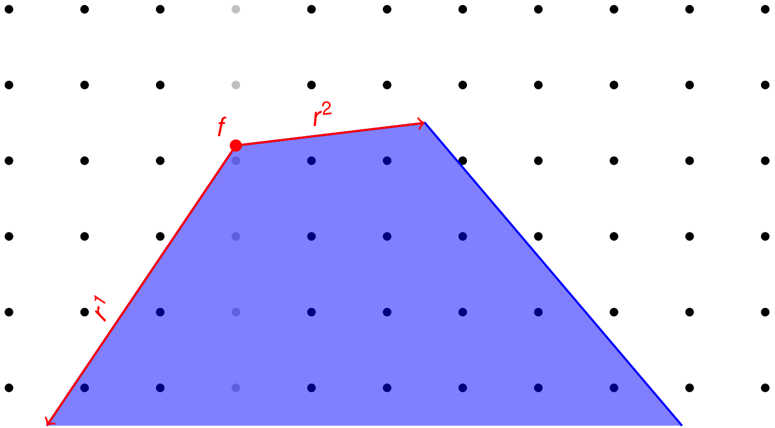

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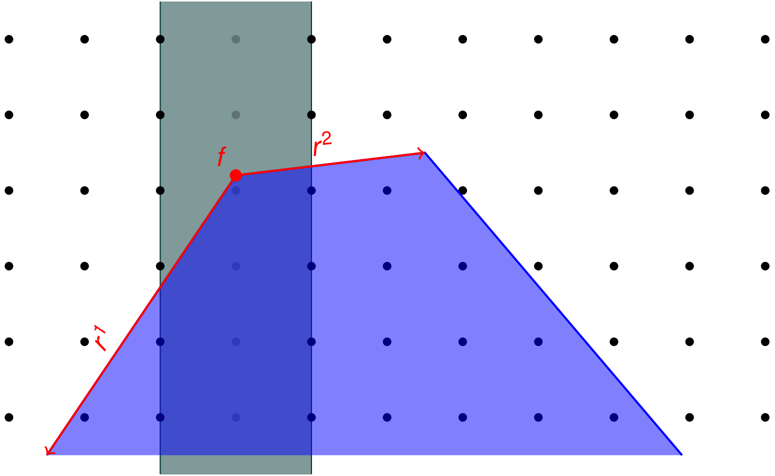

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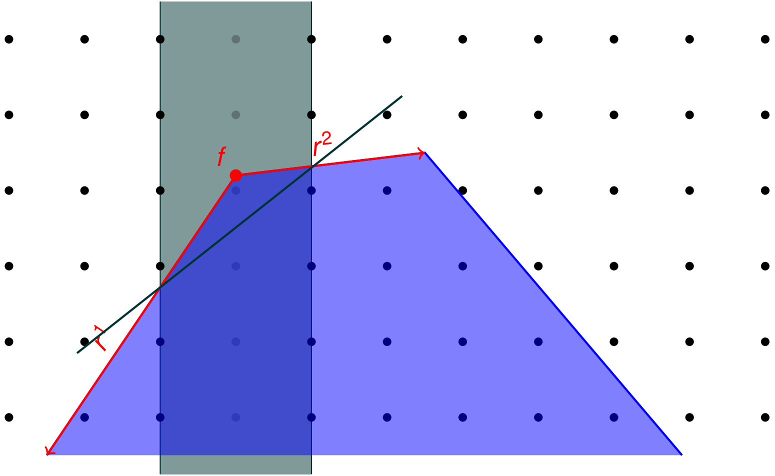

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We call these disjunctions wide split disjunctions and the derived cuts wide split cuts.

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$f_i - \lfloor f_i \rfloor$

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Outline

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Algorithm 4 pure cutting plane approach

relax integrality of (M) and solve, set $k = 1$
while ($k \leq$ n_rounds AND cuts_added) **do**
    for $i \in B \cap \{1, \ldots, p\}$ **do**
        if f_i is in a hole **then**
            compute the wide split cut and add it to (M)
        else if f_i is fractional **then**
            compute the simple split intersection cut and add it to (M)
        end if
    end for
    solve (M)
    $k \leftarrow k + 1$
end while

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	with wide splits		without wide splits	
round	#cuts (wides)	% gap closed	#cuts	% gap closed
1	62 (10)	15.11	62	15.11
2	58 (13)	18.63	57	18.63
3	62 (14)	21.24	62	21.22
4	63 (9)	22.70	63	22.52
5	62 (14)	23.89	62	23.75
6	62 (10)	24.93	63	24.89
7	63 (6)	25.53	62	25.39
8	62 (10)	26.06	62	26.01
9	62 (11)	26.38	62	26.23
10	62 (11)	26.69	62	26.27
11	60 (11)	26.87	60	26.61
12	62 (11)	27.05	62	26.62
13	62 (10)	27.16	61	26.62
14	63 (10)	27.27	62	26.62
15	62 (11)	27.34	62	26.62

$$x_i = \sum_{j \in K} \lambda_j x_j$$

$$\sum_{j \in K} x_j = 1$$

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To test the wide splits on holes that are not explicit in the model, we created random holes with 3 different intensities on 18 MIPLIB 2010 instances that contain bounded integer variables.

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instance	int_0	int_1	int_2	int_3
50v-10	39.89	39.33	40.47	40.33
germanrr	12.80	12.58	12.26	12.28
lectsched-1	0.00	0.00	0.00	0.00
lectsched-2	0.00	0.00	0.00	0.00
lectsched-3	0.00	0.00	0.00	0.00
lectsched-4-obj	50.02	75.02	100.00	75.02
mik-250-1-100-1	70.46	75.38	71.55	77.72
mzzv11	20.44	50.65	56.78	69.06
n4-3	34.00	34.00	38.55	35.36
n9-3	23.64	23.64	24.67	24.33
neos16	17.65	29.41	29.41	23.53
neos-555424	30.71	33.03	39.00	52.92
neos-686190	7.15	7.15	7.25	13.58
noswot	0.00	0.00	0.00	0.00
rococoB10-011000	13.20	19.17	13.61	20.89
rococoC10-001000	30.90	30.18	30.66	64.86
sp98ir	7.60	9.40	10.91	10.86
timtab1	33.03	34.29	34.29	34.29
mean	27,96	33,80	36,39	39,60

percentage gap closed after 15 rounds of cuts

Branch & Cut

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Assume that we have a MILP with explicit holes, modeled by the binary variables x_a,

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax = b \\ & Dx + Ex_a = g \\ & x \in \mathbb{Z}_+^p \times \mathbb{R}_+^n \\ & x_a \in \{0,1\}^k. \end{aligned} \tag{M'}$$

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- if some hole $[h_l, h_u]$ for x_i is violated, branch with h_l and h_u as new upper and lower bound, respectively:

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`CPXR+cuts`:

- the same **`CPXR`** with the separation of r rounds of wide split cuts at the root node

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We compared these three methods on TSPMTW instances from [Belhaiza 2013]. We generated four testbeds based on graphs with 10 - 40 nodes.

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	CPXF	CPXR	CPXR+cuts
testbed_1	112.33	93.47	93.39
testbed_2	92.47	115.86	114.03
testbed_3	127.80	99.66	96.83
testbed_4	176.65	145.15	161.30

average comp. times in secs

Outline

1. Introduction
2. Intersection cuts and wide split cuts
3. Computational results
   3.1 Pure cutting plane approach
   3.2 Branch & Cut
4. Further topics & Outlook

Binarization splits

Binarization splits

Let us go back to the GUB-Link type equalities:

$$x_i = \sum_{j \in K} \lambda_j x_j$$
$$\sum_{j \in K} x_j = 1$$

for some $i \in \{1, \ldots, p\}$, some subset $K \subseteq \{1, \ldots, n\}$ and $\lambda_j \in \mathbb{N}$, $j \in K$,

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In the optimal simplex tableau of the LP relaxation of (M), assume that $\lambda_\ell < f_i < \lambda_{\ell+1}$ (f_i is in a hole for x_i), and let $L = \{1, \ldots, \ell\}$. Then the split disjunction

$$\sum_{j \in L} x_j \leq 0 \quad \text{OR} \quad \sum_{j \in L} x_j \geq 1$$

is valid for (M) and violated, and takes into account the hole.

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\begin{aligned}
x_i &= f_i + \sum_{j \in N} r_i^j x_j \quad & i \in B \\
x_j &\geq 0 & j \in N,
\end{aligned}
$$

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$$\tilde{r}^j := \begin{cases} 1 + \sum_{i \in \bar{B}} r_i^j & j \in \bar{N}, \\ \sum_{i \in \bar{B}} r_i^j & j \in N \setminus \bar{N}, \end{cases}$$

then the intersection cut corresponding to the binarization split can be written as

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where

$$\alpha_j = \begin{cases} \frac{f_0}{-\tilde{r}^j} & \tilde{r}^j < 0, \\ \frac{1-f_0}{\tilde{r}^j} & \tilde{r}^j > 0, \\ +\infty & \textit{otherwise}. \end{cases}$$

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For the binarization split, the left-hand-side of the split disjunction can be written as

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All the variables in $\bar{N}$ are integer, so the lifted intersection cut obtained from the binarization split is equal to a lifted multi-row split cut, obtained from the system

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and the split convex set $S = \{x_{\bar{B}} \in \mathbb{R}^{\bar{B}} \mid 0 \leq \sum_{i \in \bar{B}} x_i \leq 1\}$.

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Thank you!