ILP formulations for finding optimal locations for charging stations in an electric car sharing network

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Introduction

Context

- **e4-share:** Models for Ecological, Economical, Efficient, Electric Car-Sharing
- Study and solve optimization problems arising in planning and operating **car sharing system** using **electric vehicles**

Electric Vehicles

- **more efficient** and **less polluting** (in urban settings)
- **shorter range** and thus frequent recharging necessary

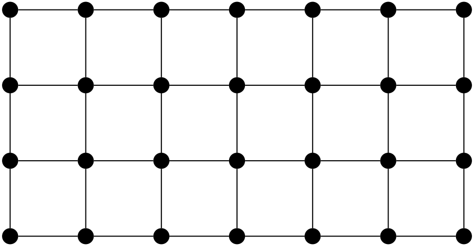
This work

- ILP formulations to find **optimal locations for charging stations**
- cars are picked up from / returned to these stations
- start and end station need not coincide.

Problem description

Problem description – Stations

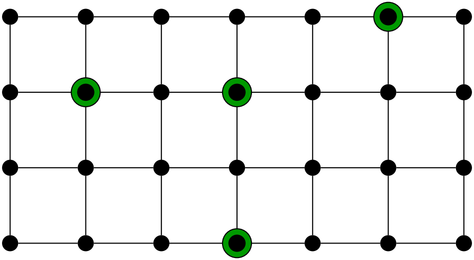
Given a **street network** $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



Problem description – Stations

Given a **street network** $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a set of potential **locations of charging stations** $S \subseteq \mathcal{V}$, where each station i has

- a **cost** F_i for **constructing** it,
- a **maximum capacity** for charging slots C_i, each of which costs q_i,



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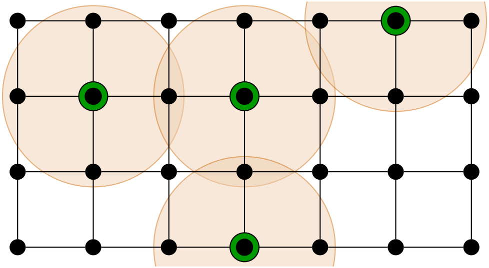

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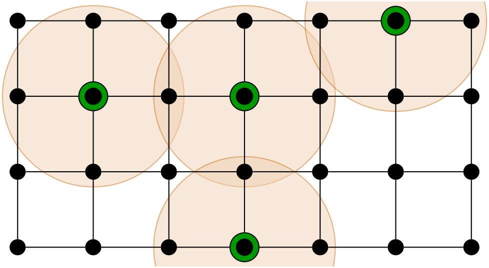
we select a **subset of stations** to be constructed, as well as their **size**, subject to a **budget constraint**.

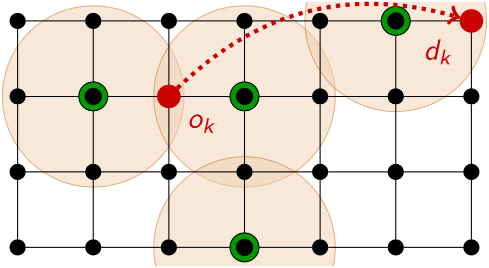


Problem description – Trips

Given a set K of requested trips, where each trip has

- **origin** o_k and **destination** d_k,
- **start** s_k and **end** e_k time,
- a **profit** p_k and
- an (over-)estimated **battery usage** b_k,

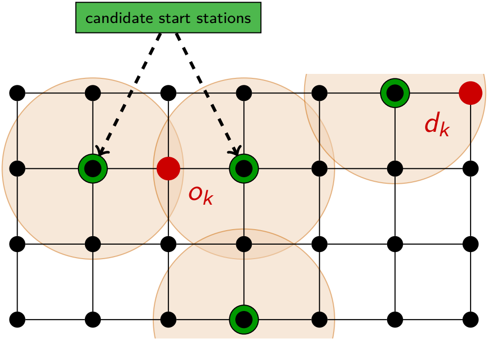
we select a **set of trips** we want to accept to **maximize** the operator's **profit**.



Problem description – Trip assignment

Each accepted trip is assigned to

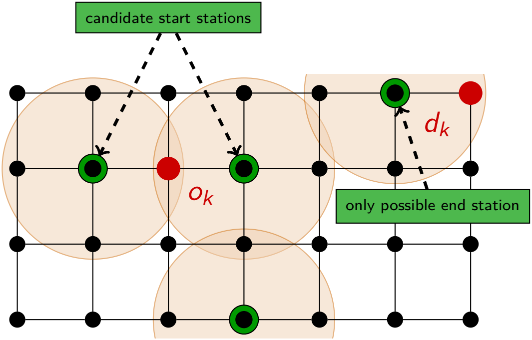
- a **start** station where the car is picked up,



Problem description – Trip assignment

Each accepted trip is assigned to

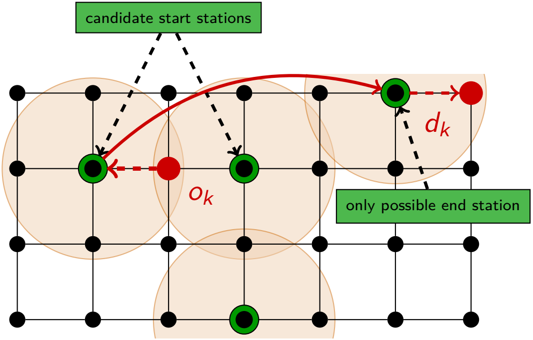
- a **start** station where the car is picked up,
- an **end** station where the car is dropped off, and



Problem description – Trip assignment

Each accepted trip is assigned to

- a **start** station where the car is picked up,
- an **end** station where the car is dropped off, and
- a car with **sufficient battery level** parked at the start station.



ILP Model

ILP model

Assumptions and Definitions

- homogeneous fleet of cars H, each costing ζ
- parked cars are recharged at fixed rate ρ
- planning horizon $T = \{0, \ldots, T_{\text{max}}\}$
- $N(v)$: stations within walking distance from v
- $\Delta_k = e_k - b_k$: the duration of trip k

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Decision variables

- $y_i \in \{0, 1\}$: whether station i is opened or not
- $z_i \in \{0, \ldots, C_i\}$: station i's assigned capacity
- $a_h \in \{0, 1\}$: whether car h is bought
- $x_k \in \{0, 1\}$: whether trip k is accepted
- $x_k^h \in \{0, 1\}$: whether car h performs trip k

ILP model

$$
\begin{aligned}
\max \quad & \sum_{k \in K} p_k x_k && && (1)\\
\text{s.t.} \quad & \sum_{i \in S} (F_i y_i + q_i z_i) + \sum_{h \in H} \zeta a_h \leq W && && (2)\\
& y_i \leq z_i \leq C_i y_i && \forall i \in S && (3)\\
& \sum_{h \in H} x_k^h = x_k && \forall k \in K && (4)\\
& \sum_{k \in K : s_k \leq t, e_k > t} x_k^h \leq a_h && \forall t \in T, h \in H && (5)
\end{aligned}
$$

ILP model

$$\max \sum_{k \in K} p_k x_k \tag{1}$$

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objective function: maximize profit of accepted trips

ILP model

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budget constraint

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stations may not exceed their maximum capacity

ILP model

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every opened station has at least one charging slot

ILP model

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$$\sum_{k \in K : s_k \leq t, e_k > t} x_k^h \leq a_h \qquad \forall t \in T, h \in H \tag{5}$$

assign every accepted trip to a car

ILP model

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a car may perform at most one trip at any time

ILP model – what's still missing?

So far, the model does **not** ensure that

- cars move along a **consistent path** throughout the network, that
- **stations' capacities** are never exceeded, or that
- a **car's battery** level never gets below zero.

We will present **two models** to enforce the first two missing aspects ("location feasibility")

- **flow model** on a time-expanded location graph
- **no-flow model**

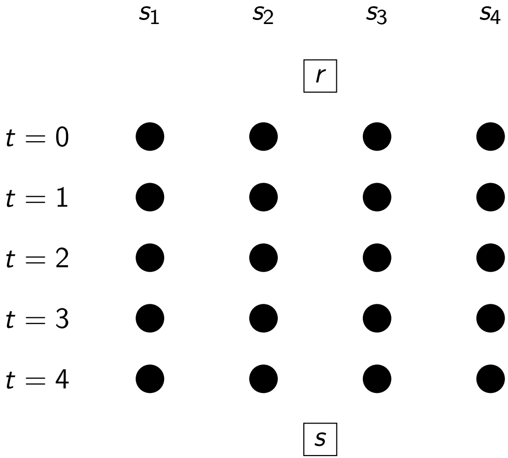
and **three models** that enforce battery feasibility

- **flow model** on a time-expanded battery graph
- **continuous** battery tracking
- battery-infeasible **path cuts**

Location feasibility

Location feasibility – Location graph

To model the location of each car at each point in time, we use a **time-expanded location graph** $G = (V, A)$.



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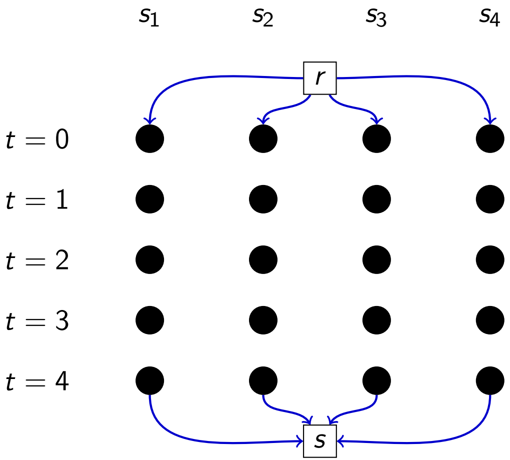
root arcs A_I and **sink arcs** A_C for initialization and collection



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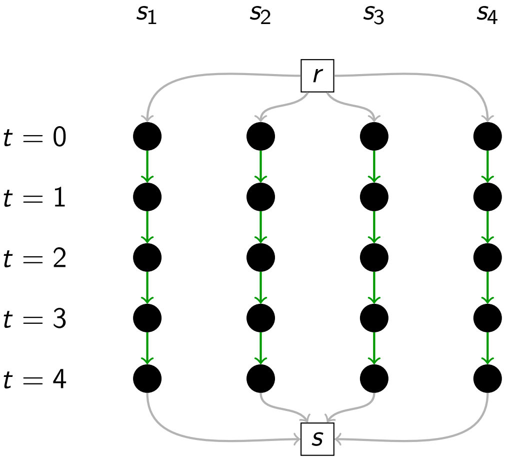
waiting arcs A_{W} for parked cars



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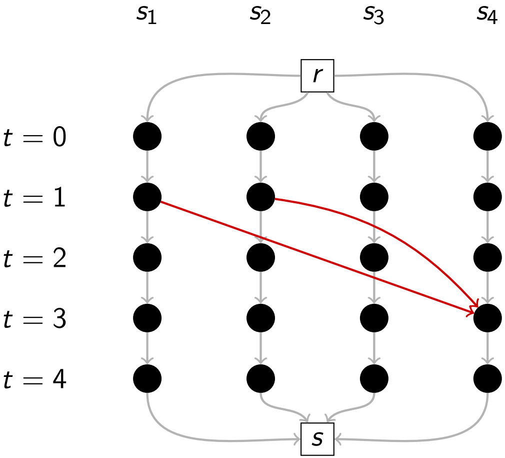
root arcs A_{I} and
sink arcs A_{C}
for initialization and collection

waiting arcs A_{W}
for parked cars

trip arcs A_{T}
for cars used for trips



Location feasibility – Location graph

Additional variables

- Flow variable $\boldsymbol{f_a^h} \in \{0, 1\}$: whether car h moves along arc a

$$\sum_{h \in H} \sum_{a \in \delta^+(i_t) \setminus A_\mathrm{T}} f_a^h \leq z_i \qquad \forall i \in S,\ t \in T \qquad (6)$$

$$f^h[\delta^-(i_t)] \leq y_i \qquad \forall h \in H,\ i \in S,\ t \in T \qquad (7)$$

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never exceed a station's capacity

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only opened stations may be used

Location feasibility – Location graph

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every bought car leaves the root

Location feasibility – Location graph

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flow conservation

Location feasibility – Location graph

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if a car performs a trip, it must move along one of its trip arcs

Location feasibility – No-flow model

Additional variables

- $\tilde{x}_k^i \in \{0,1\}$: whether trip k starts at station i
- $\hat{x}_k^i \in \{0,1\}$: whether trip k ends at station i

$$\sum_{i \in N(o_k)} \tilde{x}_k^i = x_k \qquad \forall k \in K \tag{11}$$

$$\sum_{i \in N(d_k)} \hat{x}_k^i = x_k \qquad \forall k \in K \tag{12}$$

$$\tilde{x}_k^i \leq y_i \qquad \forall k \in K, i \in N(o_k) \tag{13}$$

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assign a start and end station to each accepted trip

Location feasibility – No-flow model

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only use opened stations as start/end stations

Location feasibility – No-flow model

Additional variables

- $a_h^i \in \{0,1\}$: whether car h starts at station i

$$\sum_{i \in S} a_h^i = a_h \qquad \forall h \in H \qquad (15)$$

$$a_h^i \leq y_i \qquad \forall i \in S, h \in H \qquad (16)$$

$$0 \leq \sum_{h \in H} a_h^i - \sum_{\substack{k \in K: i \in N(o_k), \\ s_k \leq t}} \tilde{x}_k^i + \sum_{\substack{k \in K: i \in N(d_k), \\ e_k \leq t}} \hat{x}_k^i \leq z_i \qquad \forall i \in S, t \in T \qquad (17)$$

Location feasibility – No-flow model

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assign a start station to each bought car

Location feasibility – No-flow model

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only use opened stations as start stations for cars

Location feasibility – No-flow model

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number of cars parked at station i at time t

Location feasibility – No-flow model

Additional variables

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$$\sum_{i \in S} a_h^i = a_h \qquad \forall h \in H \qquad (15)$$

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ensure that capacity is never exceeded

Location feasibility – No-flow model

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ensure that no more cars leave a station than are available there

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \sum_{i \in N(o_k)} a_h^i + \sum_{\substack{k' \in K : e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h - \sum_{\substack{k' \in K : s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h \quad \forall k \in K, h \in H$$

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whether car i starts in $N(o_k)$

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \sum_{i \in N(o_k)} a_h^i + \boxed{\sum_{\substack{k' \in K: e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h} - \sum_{\substack{k' \in K: s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h \quad \forall k \in K, h \in H$$

how often car i (potentially) enters $N(o_k)$ via a trip

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \sum_{i \in N(o_k)} a_h^i + \sum_{\substack{k' \in K: e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h - \sum_{\substack{k' \in K: s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h \quad \forall k \in K, h \in H$$

how often car i (potentially) enters $N(o_k)$ in total

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \sum_{i \in N(o_k)} a_h^i + \sum_{\substack{k' \in K : e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h - \sum_{\substack{k' \in K : s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h \quad \forall k \in K, h \in H$$

how often car i leaves $N(o_k)$

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \boxed{\sum_{i \in N(o_k)} a_h^i + \sum_{\substack{k' \in K: e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h - \sum_{\substack{k' \in K: s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h} \quad \forall k \in K, h \in H$$

If this whole expression is

- ≥ 1: car i might be in $N(o_k)$
- ≤ 0: car i cannot be in $N(o_k)$

Location feasibility – No-flow model

first step to ensure connectivity: a trip k may only be assigned to a car if that car is potentially in $N(o_k)$

$$x_k^h \leq \sum_{i \in N(o_k)} a_h^i + \sum_{\substack{k' \in K: e_{k'} \leq s_k, \\ N(o_{k'}) \cap S \setminus N(o_k) \neq \emptyset, \\ N(d_{k'}) \cap N(o_k) \neq \emptyset}} x_{k'}^h - \sum_{\substack{k' \in K: s_{k'} \leq s_k, \\ N(o_{k'}) \subseteq N(o_k), \\ N(d_{k'}) \subseteq S \setminus N(o_k)}} x_{k'}^h \quad \forall k \in K, h \in H$$

If this whole expression is

- ≥ 1: car i might be in $N(o_k)$
- ≤ 0: car i cannot be in $N(o_k)$

This

- prevents many invalid trip assignments, and
- guarantees connectivity if $|N(o_k)| = |N(d_k)| = 1, \forall k \in K$.

Location feasibility – No-flow model

However, this alone is not enough in general (cars are not guaranteed to be in $N(o_k)$)
$\Rightarrow$ dynamically add additional constraints

If car h is assigned two consecutive trips k_1 and k_2 where k_2 doesn't start at the station where k_1 ends, add the following constraint

$$(1 - x_{k_1}^h) + (1 - x_{k_2}^h) + (1 - \hat{x}_{k_1}^{i_1}) + (1 - \tilde{x}_{k_2}^{i_2}) + \sum_{\substack{k \in K : s_k \geq e_{k_1}, \\ e_k \leq s_{k_2}, o_k \in N(i_1)}} x_k^h \geq 1$$

which ensure that

Location feasibility – No-flow model

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which ensure that

- car h doesn't do trip k_1

Location feasibility – No-flow model

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which ensure that

- car h doesn't do trip k_1
- car h doesn't do trip k_2

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which ensure that

- car h doesn't do trip k_1
- car h doesn't do trip k_2
- the end station of k_1 is changed

Location feasibility – No-flow model

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which ensure that

- car h doesn't do trip k_1
- car h doesn't do trip k_2
- the end station of k_1 is changed
- the start station of k_2 is changed

Location feasibility – No-flow model

However, this alone is not enough in general (cars are not guaranteed to be in $N(o_k)$)
$\Rightarrow$ dynamically add additional constraints

If car h is assigned two consecutive trips k_1 and k_2 where k_2 doesn't start at the station where k_1 ends, add the following constraint

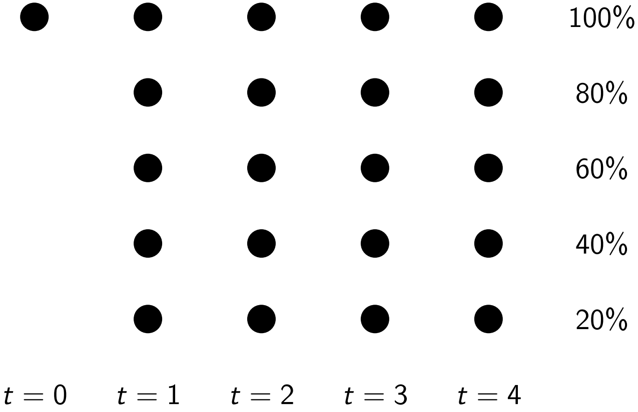
$$(1 - x_{k_1}^h) + (1 - x_{k_2}^h) + (1 - \hat{x}_{k_1}^{i_1}) + (1 - \tilde{x}_{k_2}^{i_2}) + \boxed{\sum_{\substack{k \in K: s_k \geq e_{k_1}, \\ e_k \leq s_{k_2}, o_k \in N(i_1)}} x_k^h} \geq 1$$

which ensure that

- car h doesn't do trip k_1
- car h doesn't do trip k_2
- the end station of k_1 is changed
- the start station of k_2 is changed
- car h does at least one additional trip between k_1 and k_2

Battery feasibility

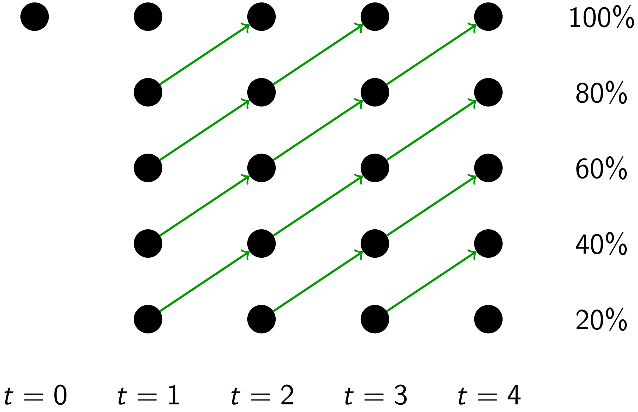
Time-expanded battery graph $G_\mathrm{B} = (V_\mathrm{B}, A_\mathrm{B})$



Battery feasibility – Battery graph

Time-expanded battery graph $G_{\mathrm{B}} = (V_{\mathrm{B}}, A_{\mathrm{B}})$

charging arcs
for parked cars

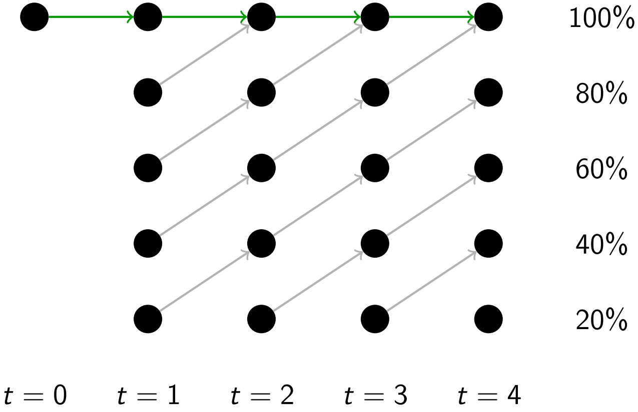


Battery feasibility – Battery graph

Time-expanded battery graph $G_{\mathrm{B}} = (V_{\mathrm{B}}, A_{\mathrm{B}})$

charging arcs
for parked cars

waiting arcs
for fully charged cars



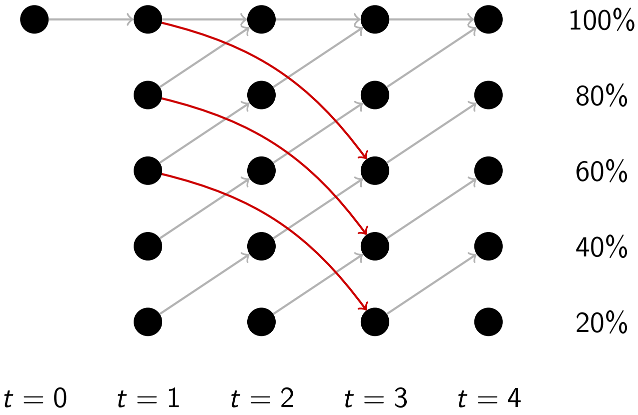
Battery feasibility – Battery graph

Time-expanded battery graph $G_{\mathrm{B}} = (V_{\mathrm{B}}, A_{\mathrm{B}})$

charging arcs
for parked cars

waiting arcs
for fully charged cars

trip arcs A_{B}^{k}
for cars used for trip k



Battery feasibility – Battery graph

Additional variables

- Flow variable $\boldsymbol{g_a^h} \in \{0, 1\}$

$$g^h[\delta^+(b_0^{\max})] = a_h \qquad \forall h \in H \qquad (18)$$

$$g^h[\delta^-(u_t)] - g^h[\delta^+(u_t)] = 0 \qquad \forall h \in H, u_t \in V_{\mathrm{B}}, 1 \leq t < T_{\max} \qquad (19)$$

$$\sum_{a \in A_{\mathrm{B}}^k} g_a^h = x_k^h \qquad \forall h \in H, k \in K \qquad (20)$$

Battery feasibility – Battery graph

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all bought cars start at battery level $b^{\max}$ at $t = 0$

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flow conservation

Battery feasibility – Battery graph

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$$g^h[\delta^-(u_t)] - g^h[\delta^+(u_t)] = 0 \qquad \forall h \in H, u_t \in V_{\mathrm{B}}, 1 \leq t < T_{\max} \qquad (19)$$

$$\sum_{a \in A_{\mathrm{B}}^k} g_a^h = x_k^h \qquad \forall h \in H, k \in K \qquad (20)$$

if a car performs a trip, it must go over one of its trip arcs

Battery feasibility – Continuous battery tracking

Additional variables

- Continuous variable $g_t^h \in [0, b^{\max}]$: battery level of car h at time t

$$g_0^h = b_{\max} \qquad \forall h \in H \qquad (21)$$

$$g_{e_k}^h - g_{s_k}^h \leq -b_k x_k^h + \Delta_k \rho (1 - x_k^h) \qquad \forall h \in H,\ k \in K \qquad (22)$$

$$g_{t+1}^h - g_t^h \leq \rho a_h \qquad \forall h \in H,\ t \in T \setminus T_{\max} \qquad (23)$$

Battery feasibility – Continuous battery tracking

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all bought cars start at battery level $b^{\max}$ at $t = 0$

Battery feasibility – Continuous battery tracking

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$$g_{t+1}^h - g_t^h \leq \rho a_h \qquad \forall h \in H,\ t \in T \setminus T_{\max} \qquad (23)$$

if a car performs a trip, its battery is depleted accordingly

Battery feasibility – Continuous battery tracking

Additional variables

- Continuous variable $g_t^h \in [0, b^{\max}]$: battery level of car h at time t

$$g_0^h = b_{\max} \qquad \forall h \in H \qquad (21)$$

$$g_{e_k}^h - g_{s_k}^h \leq -b_k x_k^h + \Delta_k \rho (1 - x_k^h) \qquad \forall h \in H,\ k \in K \qquad (22)$$

$$\boxed{g_{t+1}^h - g_t^h \leq \rho a_h} \qquad \forall h \in H,\ t \in T \setminus T_{\max} \qquad (23)$$

cars are recharged by up to ρ each time period

Battery feasibility – Battery-infeasible path cuts

explicitly forbid all battery-infeasible paths

Whenever we find a path that is infeasible w.r.t. battery consumption, we add

$$\sum_{k \in K'} x_k^h \leq f_{K'} a_h \qquad \forall K' \subseteq K, h \in H \qquad (24)$$

to the model, where $f_{K'}$ is the maximum number of trips from K' that can be performed by a single car.

Results

Computational experiments – Instances

random instances with

- grid street network
- number of stations $|S| \in \{10, 25, 50\}$
  - random location
  - random cost
  - random maximum capacity
- number of trips $|K| \in \{10, 25, 50, 75, 100\}$
  - random start and end location
  - random start and end time
  - uniform profit $p_k = 1$

We evaluated several variants of our algorithm

- **FG**: flow model with battery graph
- **FC**: flow model with continuous battery tracking
- **N**: no-flow model with battery cuts
- **NC**: no-flow model with continuous battery tracking

Computations were done with **CPLEX**, **10800 s** time limit and **3 GB** memory limit.

Improvements

Heuristic

To improve the performance of our ILP solver, we want to provide it with a good **initial solution**. We want to find a set of car paths that

- covers many profitable trips, and
- is feasible w.r.t. our budget constraints

We can find such paths by repeatedly solving the **resource-constrained longest path problem (RCLP)** on a variant of the **location graph**, where each arc is assigned

- a length ℓ_a
  - $\ell_a = p_k$ for trip arcs
  - $\ell_a = 0$ otherwise
- a battery consumption b_a
  - $b_a = -b_k$ for trip arcs
  - $b_a = \rho$ for waiting arcs
  - $b_a = 0$ otherwise

Since the location graph is acyclic, this is equivalent to solving the *resource-constrained shhortest path problem (RCSP)* on a variant where all arc lengths are negated.

Heuristic – RCLP

We solve the RCLP with a **dynamic programming labeling algorithm**. A label L consists of a profit p_L and a battery level b_L, and **dominates** L' if

$$p_L \geq p_{L'} \wedge b_L \geq b_{L'} \tag{25}$$

with at least one inequality being strict.

$labels(v) = \emptyset$
$labels(i_0) = \{(0, 100)\}, \forall i \in S$
for $t \in T, i \in S$ **do**
  for $l \in labels(i_t)$ **do**
    for $(i_t, j_{t'}) \in \delta^+(i_t)$ **do**
      if *l not dominated by any $l' \in labels(j_{t'})$* **then**
        add l to $labels(j_{t'})$
        remove all dominated l' from $labels(j_{t'})$
build car path from best label at sink

Heuristic – Algorithm

```
pathlist = ∅
while W ≥ ζ do
    W = W − ζ
    find new path with RCLP
    if W < path.cost then
        try to remove trips from path to make it feasible
    if W ≥ path.cost then
        pathlist = pathlist ∪ {path}
        W = W − path.cost
        A_T = A_T \ {a|a.trip ∈ path.trips}
        remove waiting arcs from vertices at maximum capacity
```

Symmetry breaking

Since our car fleet is homogeneous, our models have lots of symmetries. We can break these by adding constraints

$$\sum_{k \in K} \alpha_k x_k^h \geq \sum_{k \in K} \alpha_k x_k^{h+1} \qquad \forall h \in H \setminus \{H_{\max}\} \tag{26}$$

that impose an ordering on cars.
The value of a car depends on the trips it performs, such as

- their number (i.e., $\alpha_k = 1$)
- their profit (i.e., $\alpha_k = p_k$)
- their duration (i.e., $\alpha_k = \Delta_k$)

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- their duration (i.e., $\alpha_k = \Delta_k$)

Unfortunately, preliminary results are not very encouraging.

Future work

- Model extensions
  - integrating uncertainty
  - allowing car relocation
- Instances based on real world data
- Computational enhancements
  - constraint separation for fractional solutions
- Alternative formulations
  - set covering formulation (branch-and-price)